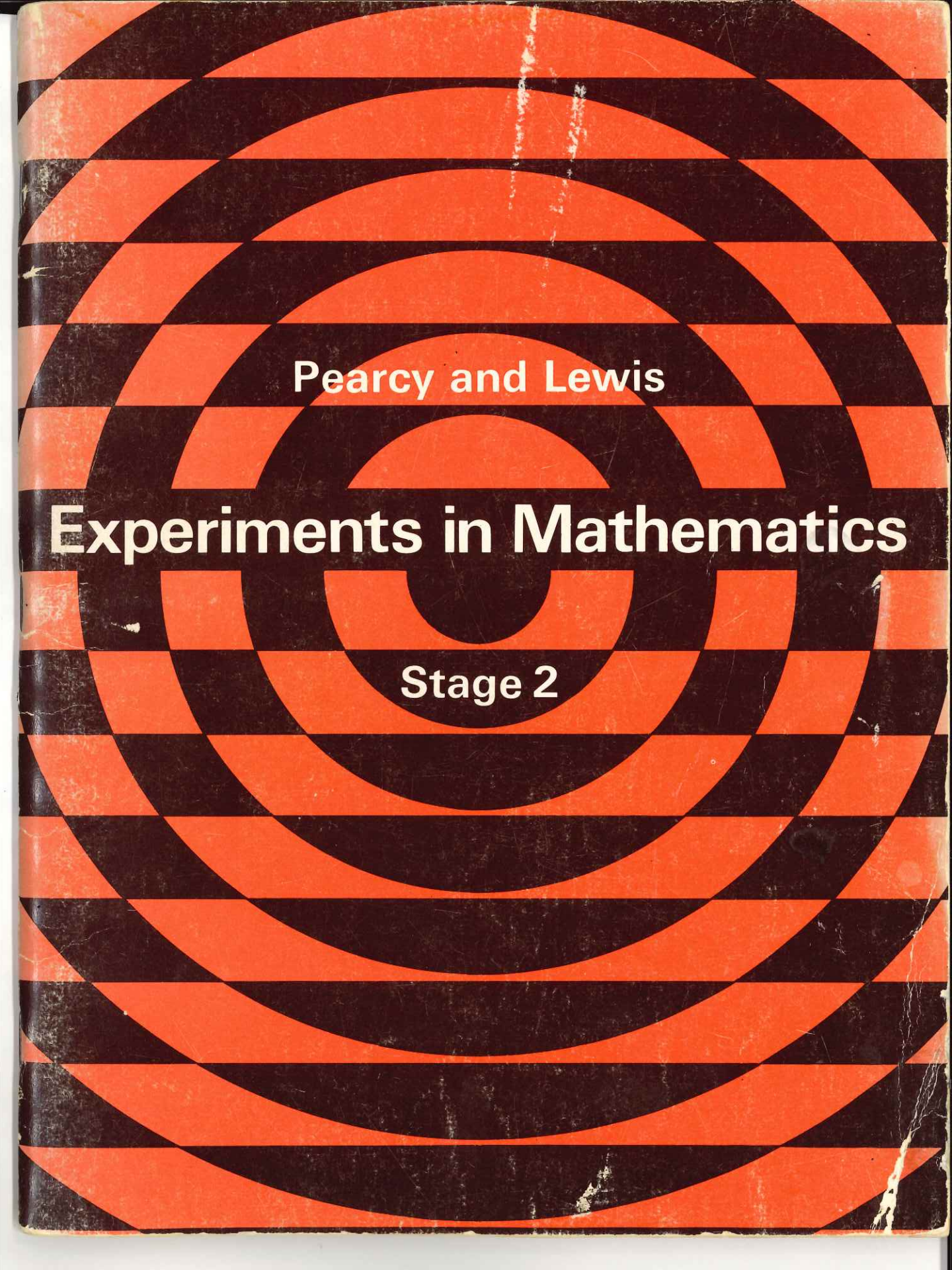




Pearcy and Lewis

Experiments in Mathematics

Stage 2

Experiments in mathematics

Stage 2

J. F. F. Pearcy

Deputy Headmaster, Holloway School

K. Lewis

Head of Mathematics Dept, Rydens School



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Introduction

These books were produced initially at the request of a group of teachers who had been provided with a Mathematics laboratory and who wanted some ideas on which to base their work. They are aimed at doing Mathematics, not just writing about it or looking at it, but actually handling, making, creating mathematical form and structure.

On every page there are experiments to be done. Some pupils will take longer than others consequently most experiments are divided into a number of parts and it is suggested that only the faster workers need do them all.

The material in each book has been found adequate to cover a year's work for a double period per week. Many experiments will occupy two or three practical sessions, especially if the pupil is encouraged to think for himself and to extend the topic. Quite a few of the actual experimental details in these booklets have been suggested by the pupils.

We are grateful for the encouragement received during the production stages from numerous primary and secondary school teachers but the main vindication of the project has been the keenness and enthusiasm of the pupils who have worked through the experiments.

J. F. F. Percy
K. Lewis

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How to use this book

1. Each pupil, or pair of pupils, should do different experiments. This keeps the quantity of special equipment needed to a minimum and also, which is perhaps more important, engenders the spirit of investigation and experiment. At first there may be a case for the whole class working together so that disciplines of recording and writing up are instilled, but too much of this will recreate the classroom situation from which these books are trying to escape.

It is therefore desirable to have a set of books which are kept in the Mathematics laboratory and which are issued to classes while in that room. A register page can be pinned up showing the pupils names and experiments which can be crossed off as they are done.

2. Pupils should be encouraged to read thoroughly the chosen experiment and to proceed with the minimum of assistance from the teacher.

3. Each pupil should be issued with an experiment folder with facilities for the inclusion of loose sheets, graph paper and work on cartridge and manilla paper. Stress should be placed on the recording of results. The display of completed tasks and of experiment folders should be a regular feature of the Mathematics laboratory.

Contents

Shapes

Transforming one shape into another, repeating shapes. Types of triangle, similarity. Combining area and arranging squares. Patterns from squares, $(a + b)^2$, $a^2 - b^2$. Patterns from square roots. The golden ratio in a triangle and rectangle. Spirals and snail shells.

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Equipment and Materials

Rulers, pencils, compasses and protractors.

Manilla paper or thin card, cartridge paper.

Coloured gummed paper. Tracing paper.

Graph paper $\frac{1}{10}$, $\frac{1}{4}$ and isometric.

Scissors.

Sellotape, paste, Seccotine.

Filter papers.

Matchsticks.

Dice.

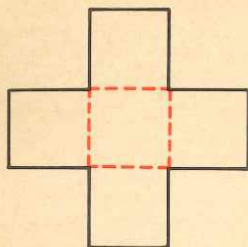
Needles and cotton.

Strawboard or Balsa wood (for slide rule, p. 32).

$\frac{1}{2}$ inch chicken wire (game of Hex, p. 53).

The Binary Adder on page 48 and the 3-D Noughts and Crosses on page 52 require more tools and material than is usual for the rest of the experiments and reference should be made to these sections.

The Greek Cross



There are many types of cross, but the Greek one is probably the simplest of all. It is made by assembling five equal squares.

The cross is of great antiquity; it was used by the Egyptians and Greeks long before the coming of Christianity. For thousands of years it was regarded as a sign of the dual forces of nature—the male and female spirit of everything that was everlasting.

Men have made this cross the subject of many dissection puzzles, a famous one is

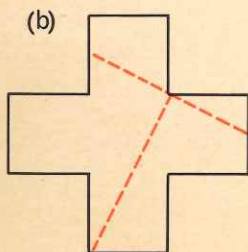
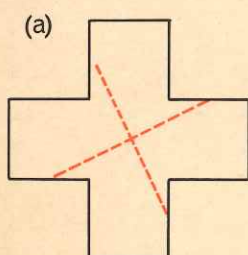
The Hindu Problem

Part 1

The problem is to cut the cross into five pieces and re-arrange them to form a square. This is supposed to be something like 3,000 years old. It appears in the seal of Harvard College and it is often given in old works as symbolical of mathematical exactitude.

Try it yourself, draw a cross on a piece of card making each square of side $1\frac{1}{2}$ inches and cut out.

Try and cut a piece off each arm so that the four pieces you cut off fit into the large remaining piece and form a square.



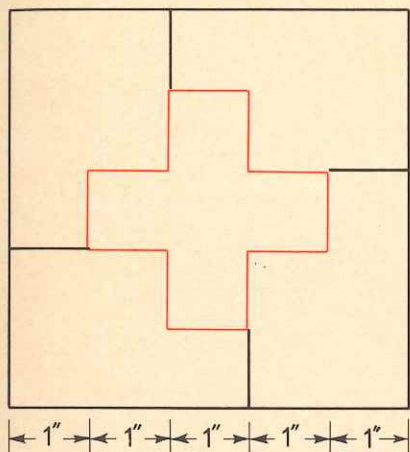
Part 2

Here are two more dissections of the same cross; notice that with the first one we have four identical pieces, and in the second one we have four differently shaped pieces, but this is obtained with only two cuts. They will each re-arrange to make squares.

Cut two more crosses and perform these dissections, do two of each and mount a 'before' and 'after' in your book.

Part 3

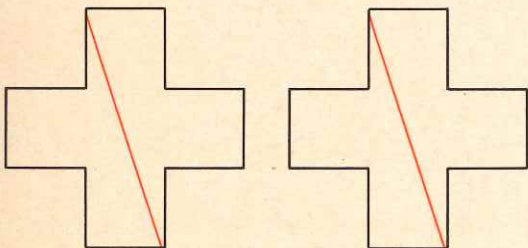
Many other puzzles exist involving this shape, for example, draw the following 'Red Cross' flag.



The four background pieces will now fit together to form another Greek cross with a swastika in the middle.

Part 4

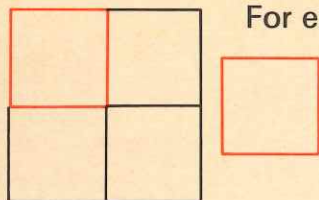
Two crosses dissect and combine to form a single square and the swastika appears once more.



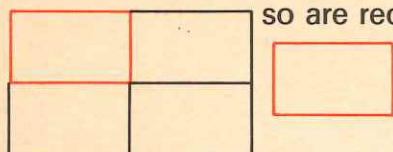
'Reptiles'

'Reptile' is really short for 'Repeat Tile'. A repeat tile is such a shape that when several are laid next to each other a similar, but of course larger shape is produced.

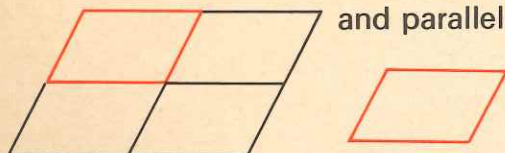
For example, all squares are 'reptiles',



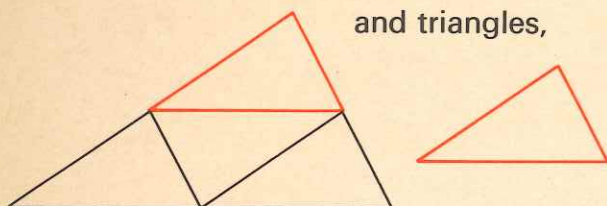
so are rectangles,



and parallelograms,



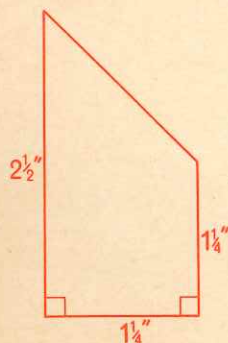
and triangles,



but in this case one of the tiles has to be turned right round.

Part 1

Here is a special trapezium which is a repeat tile. Cut four from card and find out the way in which they fit together.

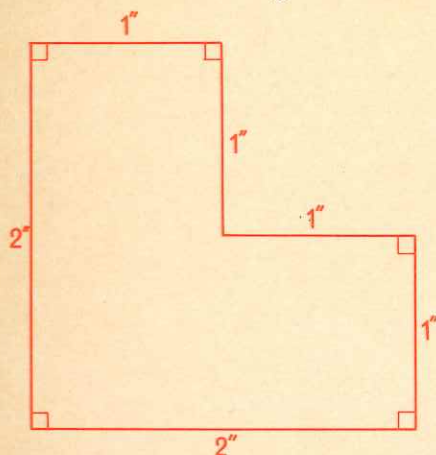


Part 2

Here are two more difficult examples.

A repeat hexagon.

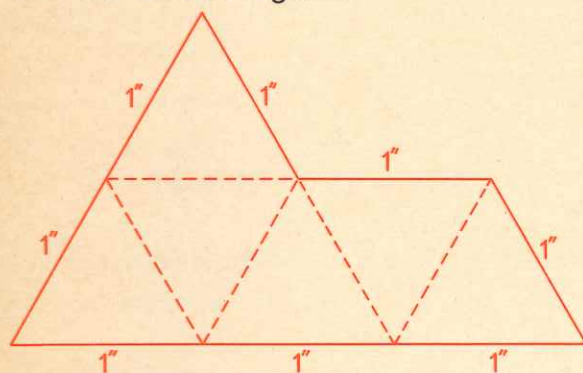
Cut four and fit together.



A repeat pentagon, sometimes called a 'Sphinx'.

See how it is composed of six equilateral triangles.

Cut four and fit together.



The Sphinx will make other shapes too. Fit two together to make a parallelogram. Fit four together to make a parallelogram.

Part 3

You should now investigate the possibility of fitting nine of any of these shapes together to make a similar figure.

Triangle Patterns

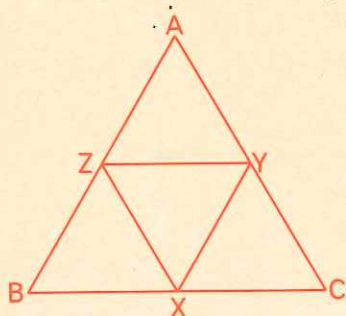
Part 1

Use cartridge paper for all these tasks.

(a) Construct this equilateral triangle of side $3\frac{1}{2}$ inches where X, Y and Z are the middles of the sides.

Cut out and crease along XY, YZ and ZX folding the vertices A, B and C upwards to meet each other.

Name the model you have made and describe it. Keep until you have finished all the other tasks and then you can mount them all in your exercise book with the results.



(b) Do just the same as before except this time start with a triangle with sides 3 inches, $3\frac{1}{2}$ inches and 4 inches. What happens when you crease and fold this one?

(c) Once more draw a triangle and make this one with sides $2\frac{1}{2}$ inches, $3\frac{1}{2}$ inches and 5 inches. Divide it into four congruent triangles as before, crease and fold.

What happens this time?

Apart from the different lengths of sides, what is the difference between the triangle you drew for part (b) and this one?

See if you can make a general statement which will summarise the results of parts (b) and (c).

(d) Finally draw a triangle with sides 3 inches, 4 inches and 5 inches.

Divide up and fold as in previous examples.

What happens this time? Is there anything special about this triangle which might be the cause of this happening?

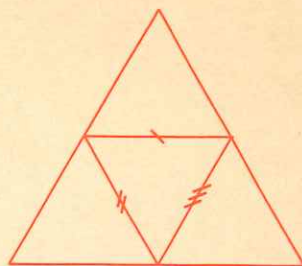
Part 2

Many other useful and interesting results can be found in this triangle pattern.

This time it would be best to use manilla paper.

Construct a triangle with sides 3 inches, $3\frac{1}{2}$ inches and 4 inches and divide up as before. Mark the sides as in the diagram and then cut the four small triangles out very carefully.

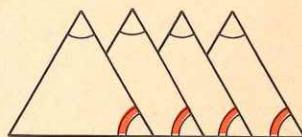
You should now be able to stack them on top of each other so that each one covers the next one perfectly and it looks as though there is only one triangle. When this happens we say that we have **CONGRUENT** triangles, they are equal in all respects.



If you spread the stack of triangles out, the equal angles can be marked.

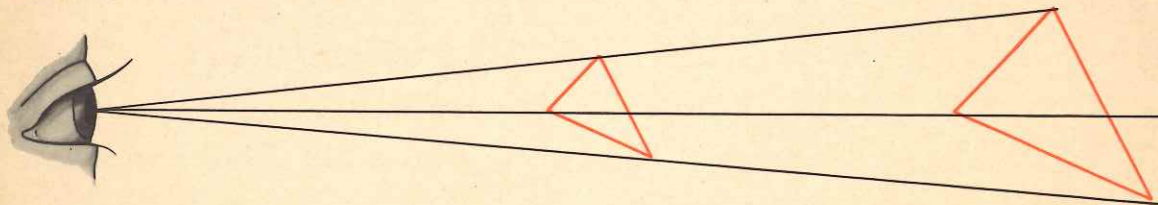
What about the third angle?

Put the four small triangles together again to make the large triangle and finish marking the equal sides. This re-assembled figure should also show you something about the sum of the angles of a triangle.



Mount and record in the usual way.

Cut two more examples of the large triangle above, mark out both. Cut the small top triangle from one of them and discard the lower part. (What is the name of this shape?)



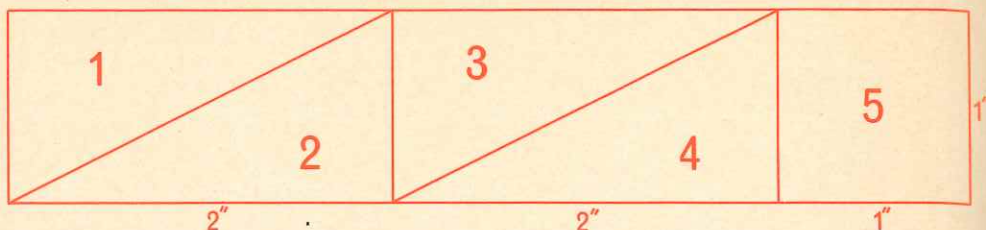
Hold one triangle in each hand, the smallest nearer to the eye and move them closer and further apart until the small triangle appears to perfectly cover the larger one. When this happens we say that the two triangles are similar.

Pythagoras' Theorem

Part 1

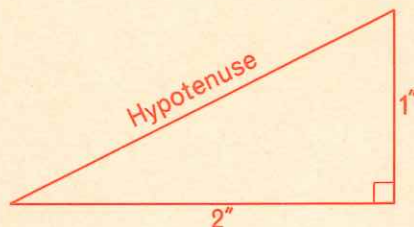
Equipment: Card, scissors and glue.

Take a rectangle of card 5 inches $\times$ 1 inch and mark it out:



Cut out the five pieces then:

- Assemble pieces 1, 2, 3 and 4 to make a square.
- Assemble all five pieces to make a square.
- Draw the following triangle in the centre of a page:



Place the square formed in (b) on the hypotenuse of this triangle.

Now cut up another strip in exactly the same way as before, place piece number 5 on the shortest side of the above triangle and the square formed in (a) on the 2 inch side. Stick these three squares down around the triangle.

The square on the hypotenuse is made up of the same pieces which go to make the other two squares on the other sides of the right angled triangle.

This shows that the area of the square on the hypotenuse is equal to the area of the squares on the other two sides of this triangle.

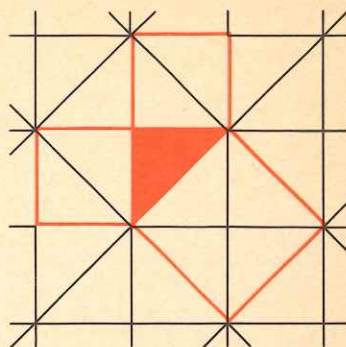
Pythagoras' Theorem proves that this result is true for every right angled triangle.

This is but one dissection to demonstrate Pythagoras' Theorem. Many others exist and it is left as an extra exercise for you to find some of them.

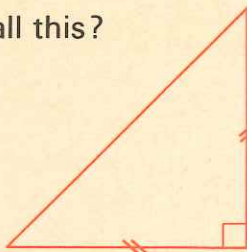
Part 2

Pythagoras was a famous Greek mathematician who lived about 540 B.C. He is said to have discovered the theorem which bears his name while looking at the tiles on the floor of an Egyptian palace.

This is a very special case where the triangle is formed by cutting a square along a diagonal.



What sort of triangle would you call this?



However, the Chinese and Egyptians knew a lot about the squares on the sides of a right-angled triangle long before Pythagoras.

The Egyptians made use of another special case to make sure that the corners of the base of the pyramids were square. The surveyors were called 'Rope Stretchers' because they used a rope with knots at intervals of 3, 4 and 5 units.

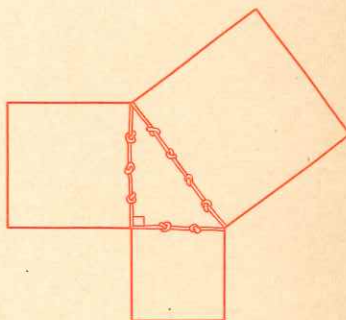
$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

Another special case is a triangle with sides in proportion 5 : 12 : 13.

Square 5 and 12 and show that their sum is 13 squared.

Draw a large accurate triangle with these sides and see that one angle is 90 degrees.



An American scientific publication has collected over 100 different proofs of this important theorem although many of them are only minor variations.

Polyominoes

Polyomino is a name recently introduced to describe a figure made by fitting more than two squares together so that they share some sides.

Domino		made of two squares
Tromino		made of three squares
Tetromino		made of four squares
Pentomino		made of five squares

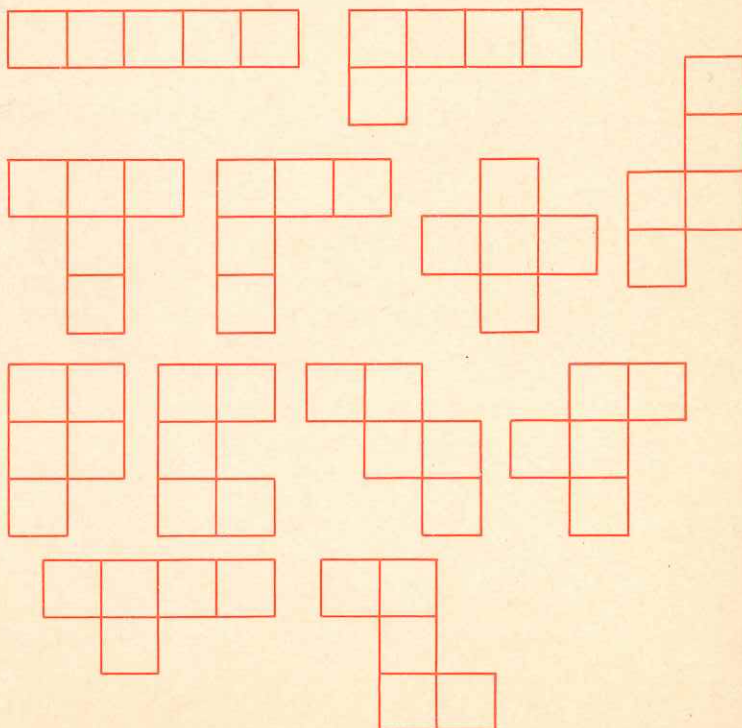
There are only two trominoes:



You can turn them over or twist them round but this does not count as a new shape.

There are five tetrominoes, see if you can draw them.

Here is the complete set of 12 pentominoes:



12 pentominoes with 5 squares each makes a total of 60 small squares.

It is possible to re-arrange these 12 pieces so that they interlock and form the following rectangles:

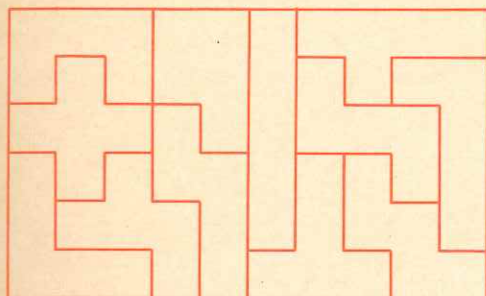
$$6 \times 10$$

$$5 \times 12$$

$$4 \times 15$$

$$3 \times 20$$

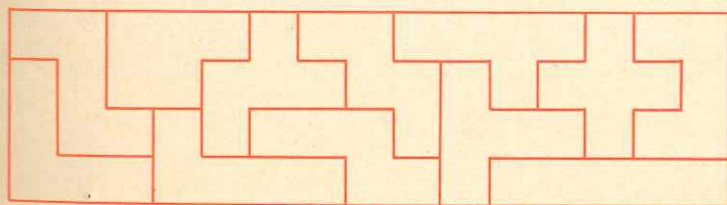
The first one, 6×10 , should be easy enough to make since well over 2,000 ways of doing it are known. The last one, 3×20 is the most difficult since there are only two ways in which it can be done.



Here is one of the 6×10 arrangements which can easily be changed into a 5×12 .

You should now make yourself a set of pentominoes and try to make some of the other rectangles. Cut them from manilla paper or card.

These 12 shapes have been made in plastic and are on sale as puzzles at toy shops.

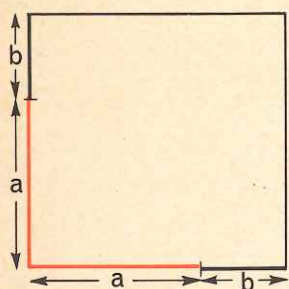


a 4×15 pattern

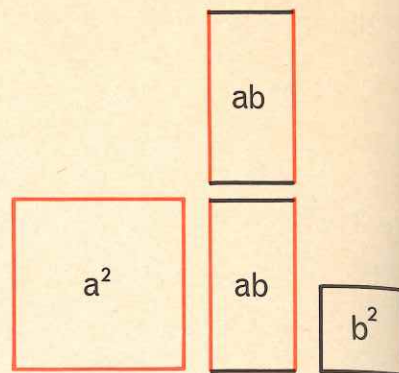
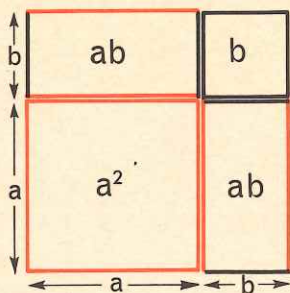
Squares

Use card to make these models and mount them in your experiment folder.

Part 1



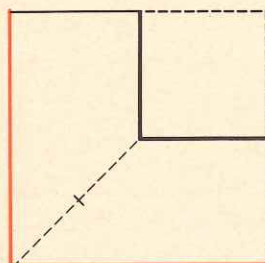
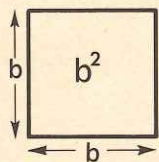
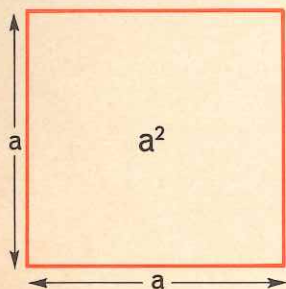
$$(a + b)^2$$



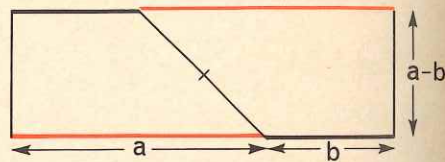
$$a^2 + 2ab + b^2$$

You should now try to make a diagram of your own to illustrate $(a + 2b)^2$

Part 2



$$a^2 - b^2$$



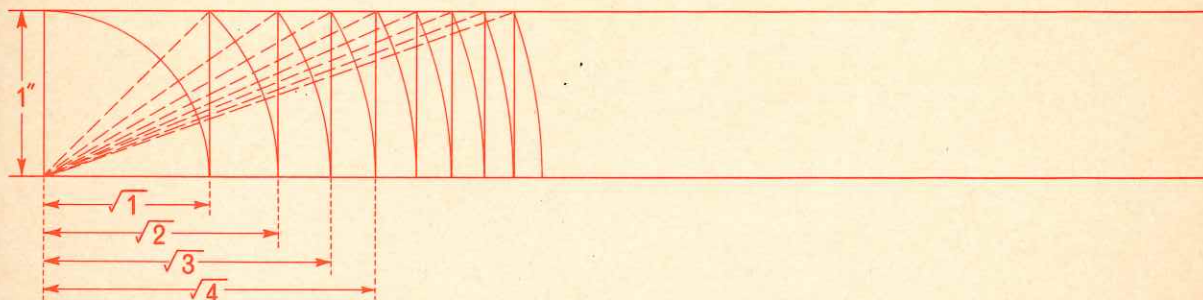
$$(a + b)(a - b)$$

This model can be made in plywood so that b^2 is hinged and can be subtracted by folding backwards and a pin through the middle of the diagonal will allow the two parts to rotate and form the rectangle.

Square Roots

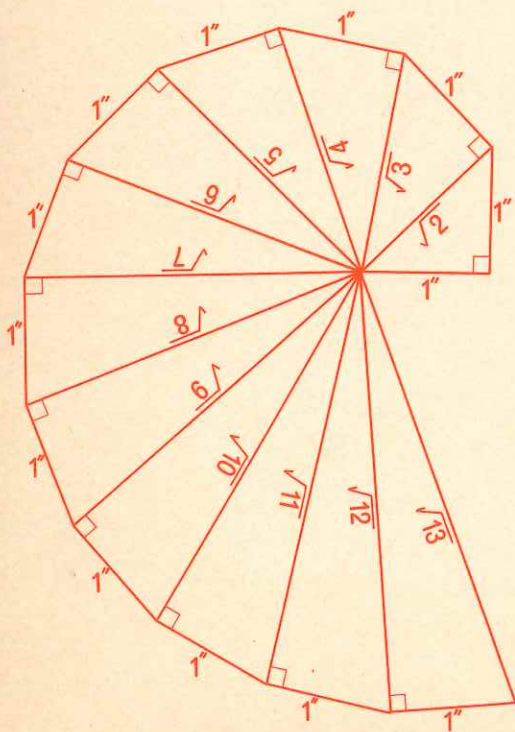
There are many ways of getting approximate values to square roots; there is an arithmetical method, you can use logarithms, a slide rule, special tables, a calculating machine or a computer. Here are two other methods which can be done by drawing.

Pattern No. 1



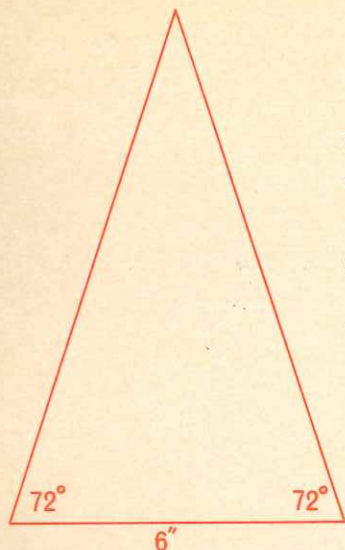
Each step gets smaller and smaller and yet it will go on for ever.

Pattern No. 2



The Golden Triangle

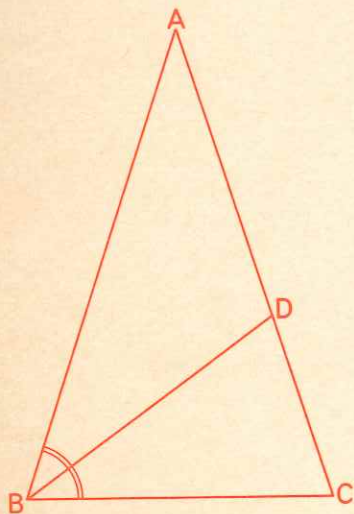
Draw this triangle on a piece of manilla paper.



Use inches and tenths to measure one of the long sides.
(Does it matter which long side you choose?)

Divide this length by the length of the short side and if you have been accurate in your drawing and measurement you should get an answer something like 1.62.

The ratio of the long side to the short side is a very special one, it is called the 'Golden Ratio' and it is approximately $\frac{8}{5}$



Bisect the bottom left hand angle of your triangle.
Measure CD.

Work out the ratio $\frac{BC}{CD}$

This should again give you the golden ratio.

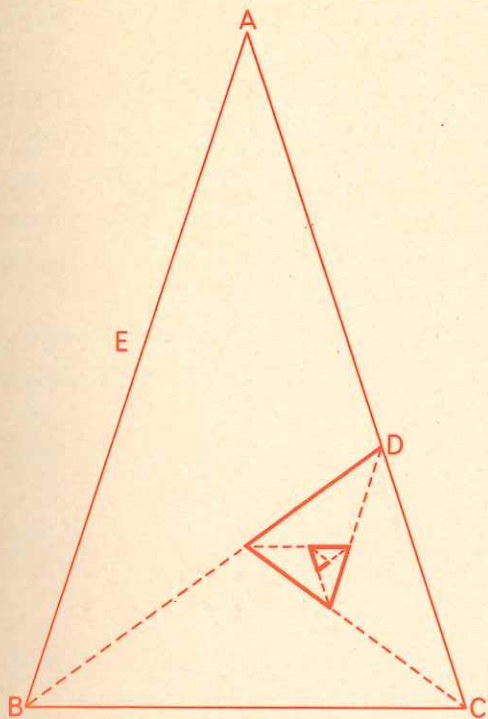
ABC is a golden triangle and so also is BCD.

What do you think would happen if you now bisected angle BCD?

Continue this process of bisecting an angle in each of the new golden triangles formed, being sure to pick the angle in the same position each time.

You will now have produced a 'nest' of golden triangles or a set of 'whirling' triangles and you can go on drawing them until your pencil is too thick to draw any more.

How many such triangles can we draw in theory?



Take a pair of scissors and cut out the original triangle. Next, starting from D cut along the thick lines. Bend each dotted line in order from the longest so that on the second fold, D touches E and continue this way until the middle is reached.

If you use balsa cement or other quick-drying glue, this folded triangle can be stuck together to make a spiral model like a triangular snail shell.

It can be painted different colours on each surface and made to look very attractive.

The golden ratio is actually found in real snail shells and in many other places in nature as well.

This is an interesting topic which you could pursue on your own. Look up 'Golden Section', 'Golden Mean' and 'Golden Ratio' in the library.

Rectangles

Part 1

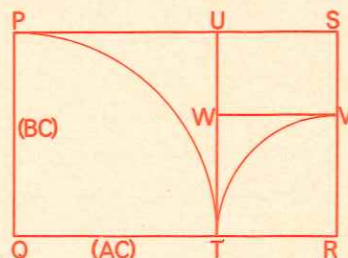
Find five or six figures in your desk or in the room which you think are rectangles and test them to make quite sure. You do not need a graduated ruler or a protractor or a set-square or a try-square, a piece of string or a straight stick will do.

We can have all sorts of rectangles, long thin ones, short fat ones and a square which is really a rectangle with all sides equal. Many rectangles seem to have sides roughly in the ratio of 8 to 5, picture frames and windows are often like this. This ratio produces a pleasing shape and such a rectangle is said to have ideal proportions.

Measure the length and the breadth of the rectangles you have discovered and divide length by breadth giving your answer to the first decimal place. The ratio of 8 to 5 is 1.6 to 1 and so you will be able to see if you have discovered any Golden Rectangles.

Part 2

Refer to the Golden Triangle in the previous experiment. Draw a rectangle PQRS with one pair of sides equal in length to BC and the other pair equal in length to AC.



This is now a Golden Rectangle.

Apart from being a pleasing shape to look at the Golden Rectangle has unusual and interesting properties.

With centre Q and radius QP draw an arc to cut QR at T and complete the square PQTU. The remaining rectangle TRSU has 'golden' proportions. With centre R and radius RT draw an arc to cut RS at V and complete the square TRVW. We have yet another golden rectangle. Continue the process with centre S and radius SV.

Carry on cutting off squares from the golden rectangles as they are formed and you will find that this will go on for ever, or until your pencil is too thick to draw any more.

There is a spiral hidden in this pattern of rectangles, see if you can find it.

Part 3

Let us see if you can discover something else about rectangles.

First of all decide on five rectangular shapes, the ones which were used for part 1 will probably do. Do not choose anything large or else the calculations might become too unwieldy.

Make sure that you have the measurements in inches and tenths of the length (L), breadth (B) and the diagonal (D) of each one.

We use inches and tenths so that it is easy to write results as decimals. If you used eighths, twelfths or sixteenths you would have to do extra work.

Enter the results in the table.

Name of rectangle	L	B	D	L^2	B^2	D^2	$L^2 + B^2$

You will have to work out the figures to enter into the last four columns, remember

$$L^2 \text{ means } L \times L$$

e.g. if the length of a side is 1.7 inches then L^2 is 1.7×1.7 which comes to 2.89 square inches, and we should round this off to the first place of decimals and enter it in the table as 2.9.

Examine the last two columns and see if you can notice anything about them.

This is really another way of looking at Pythagoras' Theorem.

Codes

Part 1

Mathematics has played a great part in the construction and solution of secret messages. Often very simple arithmetical processes will effectively disguise a message for a short while, for example take the message:

'Meet me behind the old slaughter house'.

A count will reveal that this contains 32 letters and these can therefore be arranged in rectangles:

16×2 , 2×16 , 8×4 , 4×8

(What would 32×1 and 1×32 look like?)

Consider an 8×4 arrangement

M	E	E	T	M	E	B	E
H	I	N	D	T	H	E	O
L	D	S	L	A	U	G	H
T	E	R	H	O	U	S	E

this could then be transmitted by columns from the left

MHLTEIDEENSRTDLHMTAOEHUUBEGSEOHE

and as long as the person receiving the message knows what you have done, he can read it in a matter of seconds.

Write the above message as a 2×16 rectangle and transmit in columns from the left. Does this disguise the message just as much as the 8×4 ?

What do you think you could do if your message did not have a nice lot of factors as 32 did, or worse still, what if the total came to a prime number?

In the above examples we transmitted the rectangles in columns from the left, we could if we liked, have transmitted in columns from the right, or up and down, or diagonally, in fact a 6×4 rectangle has 40 possible routes. See how many you can find in your 8×4 .

Part 2

Another way of sending a secret message is by substitution and use of a code word.

If our code word is 'Education' then we write it under the alphabet:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
E	D	U	C	A	T	I	O	N	B	F	G	H	J	K	L	M	P	Q	R	S	V	W	X	Y	X

after the code word we write the remaining letters of the alphabet, omitting those we have already used.

Our message about the slaughter house will then be:

HAAR HA DAONJC ROA KGC QGESIORAP OKSQA

and it would be wise to send this in one long string and let the decoder sort the words out rather than the enemy

HAARHADAONJJCROAKGCQGESIORAPOKSQA

This type of code can be broken by considering letter frequency, the most recurrent letters in the English language are:

E T O A N I R S H . . .

If a sufficiently long message is obtained encoded from ordinary English then the chances are very high that the most frequent letter is really an E, the next most frequent T, then an O and so on.

In practice the task is quite difficult and tables of the most frequent singles, doubles, initials, finals and many other combinations are available.

Part 3

Count the number of times each letter of the alphabet occurs in the following passage from *In the Wet* by Nevil Shute.

'He said something that neither of us could quite understand, and then turned and went back into the house. We stayed on the verandah watching the animals for a time, and because Liang was not with us I could speak more freely to the sister. "What's the position with Stevie?" I asked her. "Do you think he'll get through?"'

List the top six in order and compare your result with the known frequency of the language given above. How would you explain any differences?

Now count all the words with 1 letter, 2 letters, 3 letters and so on.

See if you can represent these results graphically.

Dice

For these experiments three dice will be needed.

In the first book of experiments it was stated that the five regular polyhedra (The Platonic Solids) could all be used as dice. See if there is a set in the room which you can study while reading what follows.

In each case every face has an equal chance with every other face of appearing uppermost when rolled. With the exception of the cube, solid regular polyhedra are difficult to make accurately and this must be one of the reasons why we do not see dice made in this shape. Some of them will not roll properly and others tend to roll too much. If we used a regular tetrahedron as a four-faced die which face would be counted?

The ordinary die is in the shape of a cube, it rolls quite well, is easy to make accurately and there is no doubt about which face is uppermost. Have you noticed that opposite faces add up to seven? Check up with your dice and see that this is so. Can you see any reason for this?

Part 1

If you rolled one die 60 times how many 6's would you expect to get? How many 3's?

Roll a die 60 times and complete the table of results:

Score	1	2	3	4	5	6
No. of times out of 60						

What do you notice about the results?

Plot your results on a graph and also plot on the same graph what you consider to be a 'perfect' set of results.



Part 2

Let us now consider what happens when two dice are rolled.

For this it is best to work in pairs, one to roll the dice and the other to record the scores by a tally system.

What do you think is the easiest score to obtain with two dice? If you rolled the pair of dice 180 times how do you think the scores would be distributed?

Roll a pair of dice 180 times and complete the table.

Score											
No. of times out of 180											

Plot a graph as before and compare your results with the 'perfect' set of results given in the answers.

Part 3

Finally let us consider what happens when three dice are rolled.

For the results to have any value a large number of rolls will be needed, the best way to do this is to get many pairs all doing the same experiment and to pool all the answers.

Each pair to roll three dice 216 times. Make a table of the combined results and plot a graph as before.

Which score was the easiest to obtain this time?

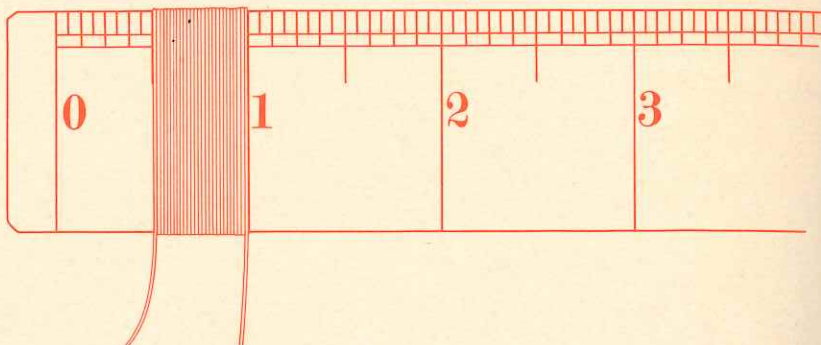
Look at the answers for a 'perfect' set of results.

Accurate Measurement

Part 1

To find the Thickness of a piece of Cotton

Wind the cotton carefully and evenly around a ruler, making sure that there are no overlaps or gaps. Carry on until at least half an inch has been covered. By counting the number of pieces of cotton which when placed side by side cover a known distance you should be able to give a fairly good estimate of the thickness of the cotton.



Would a more accurate result be obtained by covering more than $\frac{1}{2}$ inch of the ruler?

You could use this method to estimate the thickness of a human hair; only a slight modification would be needed. Try and decide how it can be done and then obtain as long a hair as possible and measure its thickness.

Part 2

If you wanted to discover the thickness of a page of a book a good way would be to measure a large number of pages and take the average.

What do you think would be a good number of pages to take, and also what do you think would be a good unit to measure in?

Which book in your desk has the greatest number of pages? Obtain an estimate for the thickness of one page.

(Try and make sure that all the pages you measure are the same thickness by eliminating special pages at each end etc.)

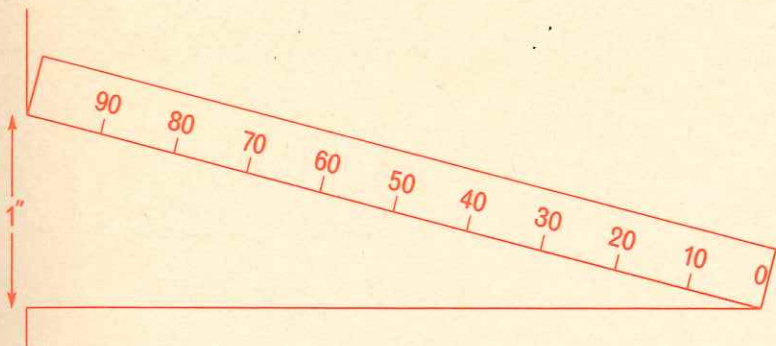
Part 3

To measure to the nearest Hundredth of an Inch

Take a thick piece of card about 3 inches $\times$ 5 inches.

A triangular notch has to be cut in this and the sloping side (the hypotenuse) has to be divided into 100 equal parts.

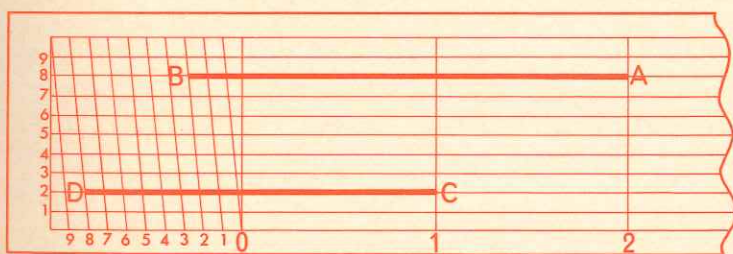
A suitable scale for this is millimetres, in fact if you have an old broken ruler you could use part of the actual scale.



See if you can discover how to use this scale and try it out on various objects.

This is a practical use of a piece of geometry known as the Intercept Theorem.

The same principle is used in a Diagonal Scale which is found on some rulers.



Line AB is 2.28 inches long.

Line CD is 1.82 inches long.

Can you see how this is obtained? Try and measure other lines and objects of your own. If you do not already have a ruler with this scale on you should try to make one and keep it for future use.

Devices for Addition

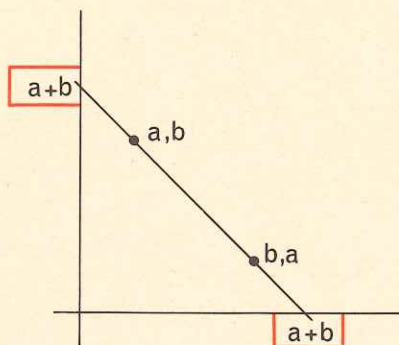
Part 1

On a piece of graph paper mark a pair of straight lines intersecting at right-angles near the bottom left hand corner.

Number up to 12 in the same equal steps along each, call the horizontal line the AXIS OF X and the vertical line the AXIS OF Y.

To add 2 and 7: Plot the points $x = 2, y = 7$
and $x = 7, y = 2$

join these two points with a straight line and produce it in both directions so that it cuts both axes. The answer, 9, should appear on either axis.



Flow diagram

Try the following additions (i) $3 + 7 = 10$
(ii) $6 + 5 = 11$
(iii) $2 \cdot 4 + 5 \cdot 7 = 8 \cdot 1$

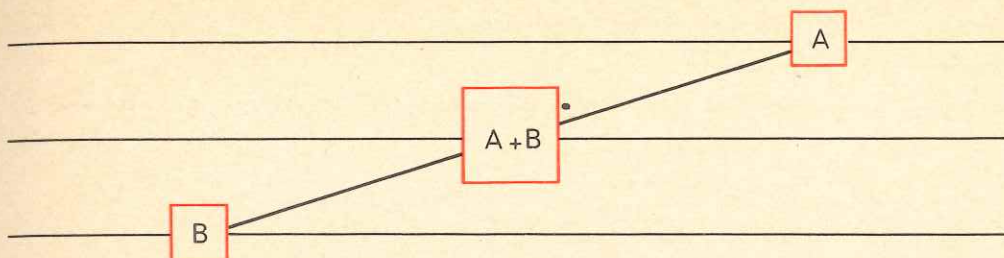
Part 2

Here is another device:

Copy the scale on a piece of card.

0	1	2	3	4	5	6	7	8	9	10	11	12
0	2	4	6	8	10	12	14	16	18	20	22	24
0	1	2	3	4	5	6	7	8	9	10	11	12

This Flow Chart shows you how the device can be used for addition.



This sort of arrangement is called a **NOMOGRAM**.

Carry out the following sums by means of your nomogram: (a) 12 plus 4 (b) 6 plus 9 (c) 7 plus 7.

This same nomogram can be used for subtraction, find out how this can be done by trying these sums:

(a) 16 minus 9 (b) 20 minus 12 (c) 11 minus 11.

Try and construct the flow chart to show this process of subtraction.

Part 3

A similar nomogram can be used to do addition and subtraction in bases other than 10, for example:

A Base 6 Nomogram for Addition:



Now try and construct a base 4 nomogram for addition of your own.

Napier's Bones

Equipment: Card, scissors, glue.

Part 1

Draw the following on card and cut along the thick lines to produce 11 vertical strips and one horizontal strip.

I.S. 1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	1	1	1	1	1
3	0	3	6	9	1	1	1	2	2	2
4	0	4	8	1	1	2	2	2	3	3
5	0	5	1	1	2	2	3	3	4	4
6	0	6	1	1	2	3	3	4	4	5
7	0	7	1	2	2	3	4	4	5	6
8	0	8	1	2	3	4	4	5	6	7
9	0	9	1	2	3	4	5	6	7	8

Take a piece of card 5 inches $\times$ $5\frac{1}{2}$ inches and stick the plain horizontal strip along the bottom and the left hand index strip (I.S.) which is numbered from 1 to 9 in its position on the left. You will have now made a cardboard model of Napier's Bones, an old calculating device.

To perform the multiplication sum $3 \times 23 = 69$.

Select the 'bones' headed 2 and 3 and place them next to each other on the base-board next to the index strip, you should then be able to read the answer straight off.

$$2 \times 78 = 156$$

Can you see how this is obtained from the bones?

Try the following sums:

- (a) 2×34 (b) 7×76 (c) 5×45 (d) 7×208 (e) 6×44
 (f) 24×57 (g) 46×67

John Napier was a Scotsman born in 1550 and he made considerable contributions to the art of calculating. His most important claim to fame is his invention of logarithms in 1614. He died in 1617 and his last work included a description of what is known as Napier's Rods or Bones.

Why do you think they are called bones?

1	7	8
2	14	16
3	21	24

Part 2

Napier's bones can still be put to good use today when we are dealing with numbers to bases other than 10. For example, it is no good trying to do a base 5 multiplication sum on a machine or with log tables, but we could make a set of bones to do the job:

1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	3	22	31

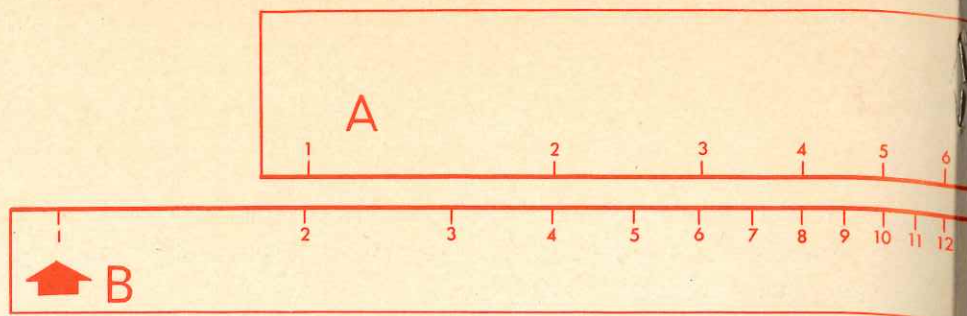
Make a set and try these sums:

- (a) 2×12
 (b) 4×24
 (c) 3×301 all in base 5
 (d) 4 squared
 (e) 32×43

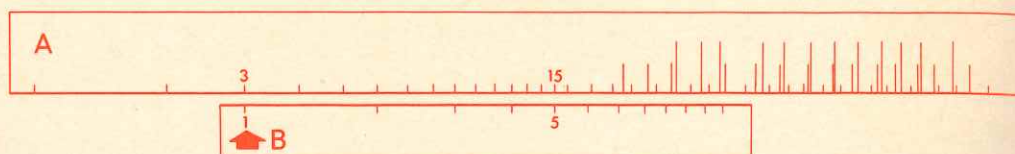
Now make yourself a set of bones to work in base 8.
 What would a set of base 2 bones look like?

The Slide Rule

Carefully copy scales A and B onto two pieces of card.

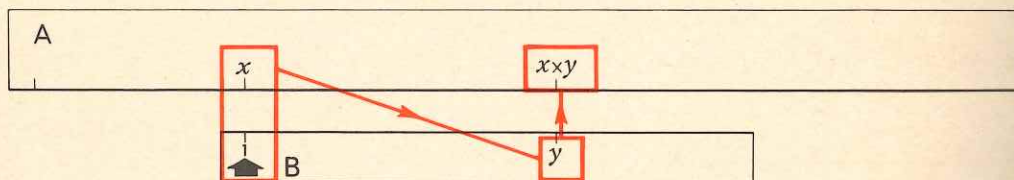


To multiply two numbers together, for example, 3 times 5:

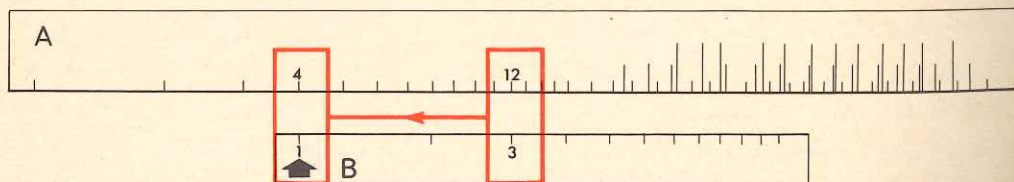


Set the arrow of the B scale under the 3 of the A scale. The slide rule will now be multiplying by 3. See what number is above the 5 of the B scale.

Practice using the rule on all the multiplication tables. Here is the flow diagram:

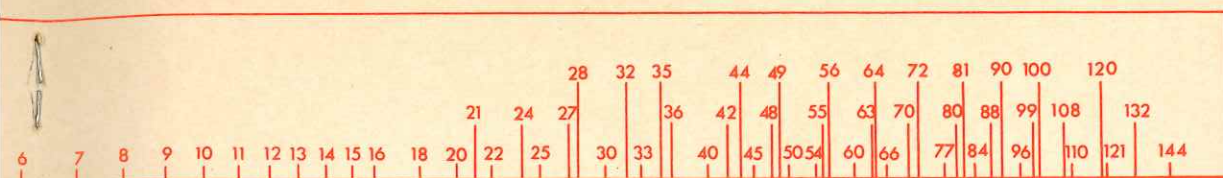


To divide one number by another, for example $12 \div 3$:



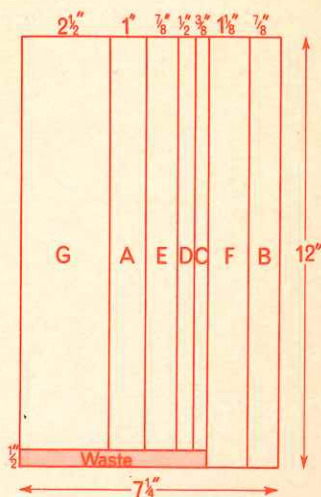
This is just the reverse of multiplication.

After some more practice see if you can mark in accurately some extra points, e.g. $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $1\frac{1}{4}$, $1\frac{3}{4}$ and so on.



To make your own slide rule all that is needed is a piece of card or strawboard and some glue. If you have balsa wood that would be better still.

The plan shows how to cut all the pieces from a rectangle $7\frac{1}{4}$ inches by 12 inches.



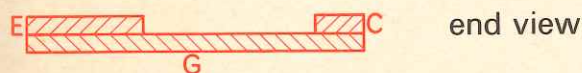
1. Glue B on top of F so that there is an equal margin on each-side. This is the slider.



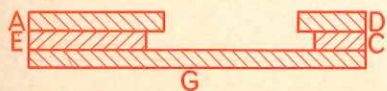
2. Glue E flush with one side of G.



3. Place slide on top of G and against E; use as a spacer to locate and glue C. Make sure that there is a small gap each side of the slide.



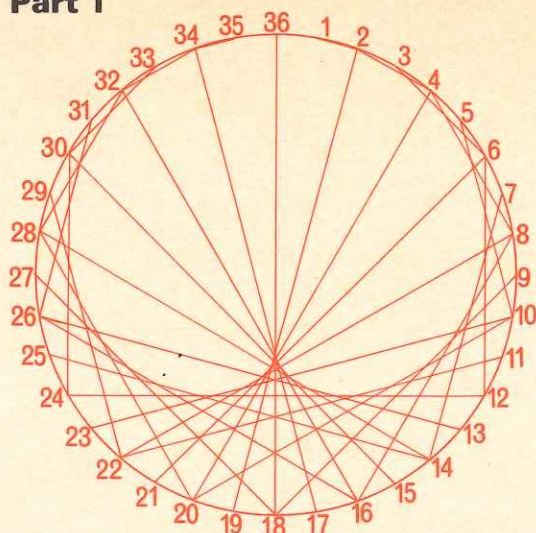
4. Still using the slide as a spacer glue on pieces A and D.



5. Stick scales onto A and B.

Envelopes

Part 1



Draw a circle on card or manilla paper and divide the circumference into 36 equal parts. Number the points lightly in pencil from 1 to 36 and prick holes through with the point of a pair of compasses.

Each point is to be joined with a line to a point which is twice its number, i.e.

1 is joined to 2

2 is joined to 4

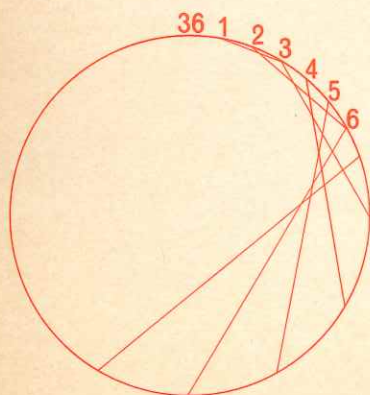
3 is joined to 6

and so on.

What are you going to join point 19 to?

When finished all these lines envelop a curve called a **CARDIOID**. Find out how it gets this name.

Rub out the numbers.



Part 2

Prepare a card in exactly the same way as you did before.

This time draw a line from every point to a point which is three times its number, i.e.

1 is joined to 3

2 is joined to 6

3 is joined to 9

and so on.

The curve which appears this time is called a NEPHROID which means shaped like a kidney.

You could continue this process with other cards, joining 1 to 4, 2 to 8, 3 to 12 . . .

The cardioid and nephroid can also be obtained from the envelopes of a set of circles.

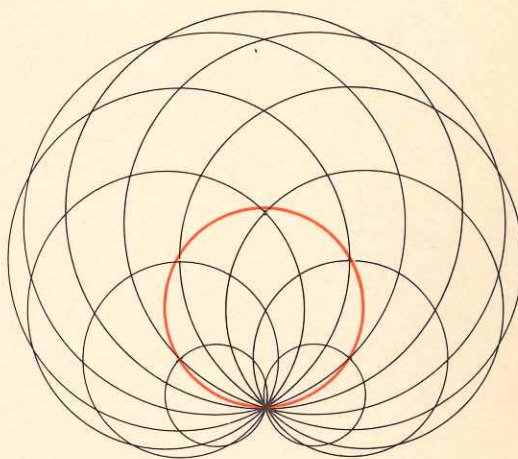
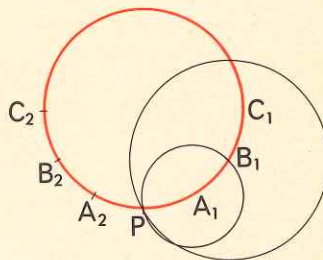
Part 3

Draw a base circle and mark off 12 points evenly spaced around the circumference.

With centre A_1 draw a circle to pass P. Keep the radius the same and draw the same circle with centre A_2 .

Centre B_1 draw a circle to pass through P and draw the same circle with centre B_2 .

Continue drawing circles to pass through P from the other points around the base circle.

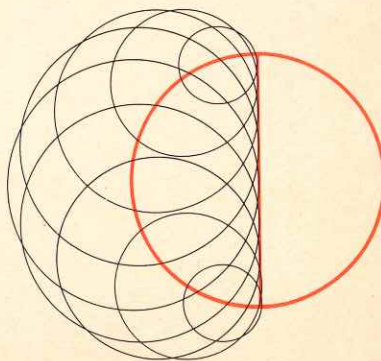


Part 4

Draw a base circle and mark off 20 points evenly spaced around the circumference.

Put in a diameter.

Draw circles with centres at the points on the base circle and such that each one just touches the diameter.



Ellipses

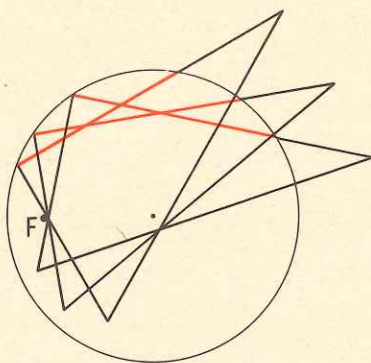
In the first Book of Experiments we saw how to draw an ellipse with two drawing pins, a loop of string and a pencil, we shall now see other ways of making this shape.

Part 1 An envelope of circle chords

Draw a circle and mark a point F fairly near to the edge.

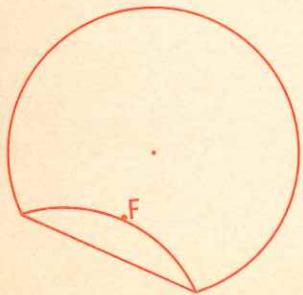
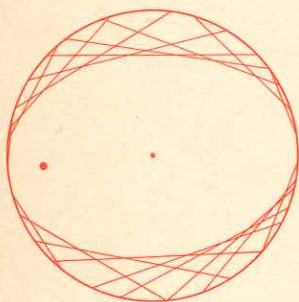
Take a set square and place it so that one side touches F and the corner of the right angle lies on the circle.

Draw the chord of the circle made by the other side and continue for numerous positions of the set square, the more you do, the better will be the final result.



Here is the completed envelope. Can you find another point which matches F?

What happens as the position of F varies, what if F was in the centre of the circle?



Part 2 An envelope by folding

Take a circle of paper and on it mark a point F fairly near to the edge. (Filter papers are very useful for this).

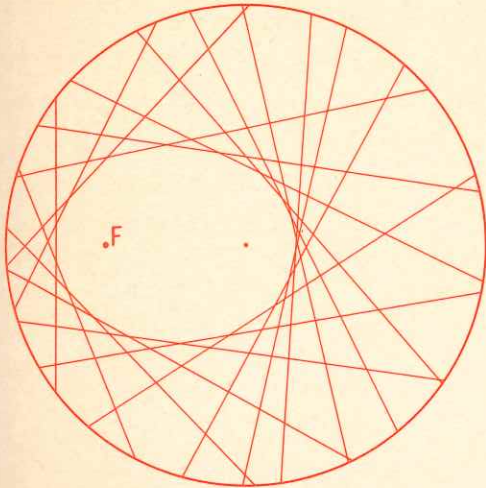
Fold the circle over so that the circumference just touches the point, unfold and go over the crease with a pencil.

Continue folding and marking until the envelope of the ellipse becomes plain.

The completed envelope.

Notice that in this case the ellipse is not symmetrically placed within the circle.

What happens as F moves this time?

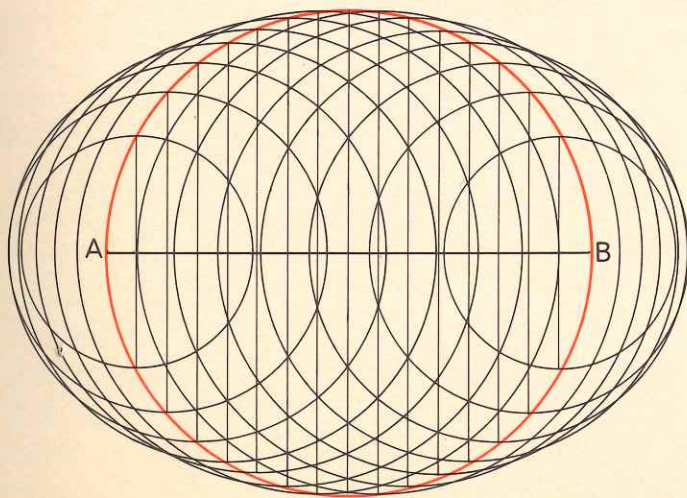


Part 3 An envelope of circles

Draw a base circle of radius 4 cm and mark off every 5 mm along a diameter AB. At these points erect chords perpendicular to AB.

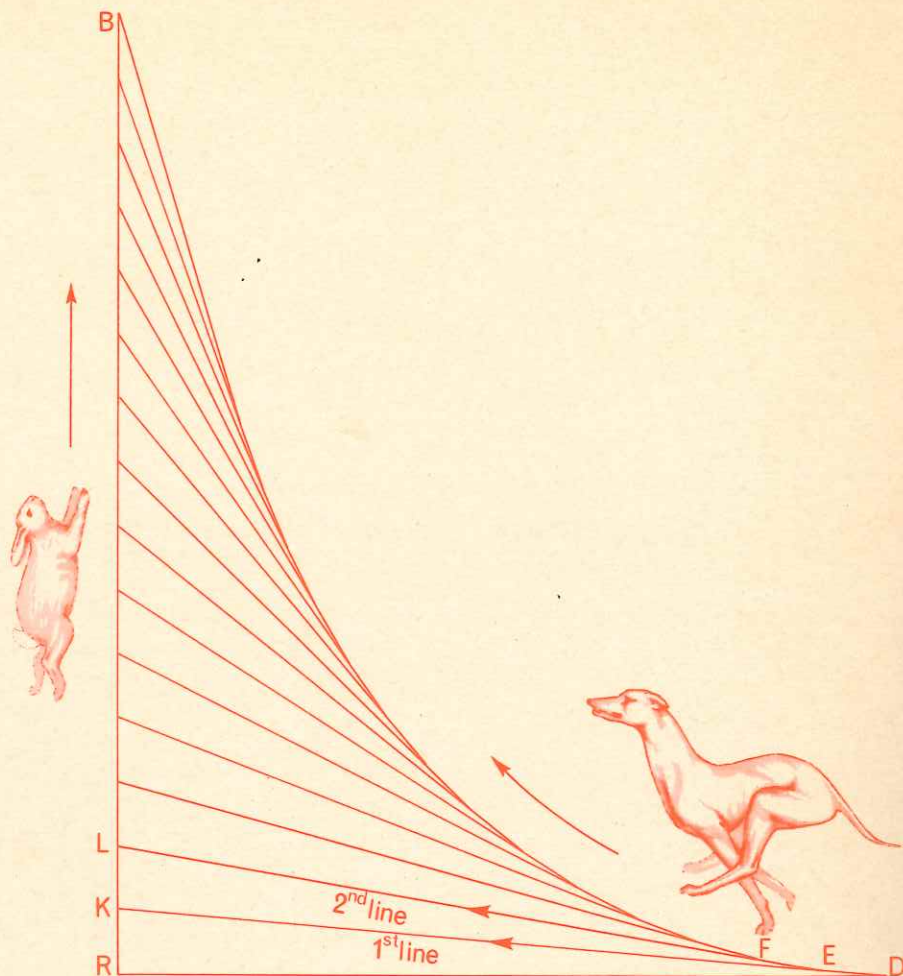
With each chord as diameter draw circles until the envelope of the ellipse is obtained.

Note that not all the circles are needed. Why not? At what point do they become unnecessary for the envelope?



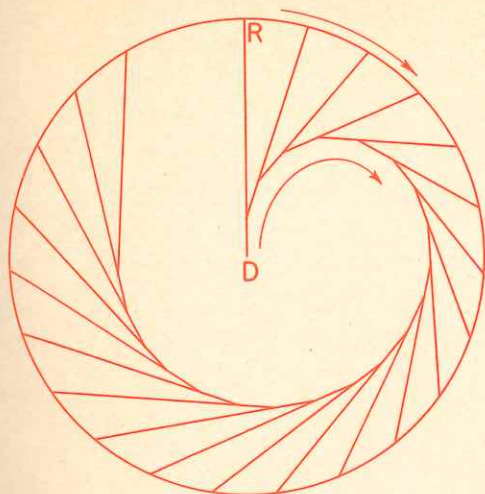
Curves of Pursuit

Interesting envelopes of curves can be obtained by considering the path of a dog chasing a rabbit.



Imagine a rabbit standing at R and a dog at D. The rabbit starts to run along the line of the hedge to its burrow at B. When the rabbit reaches point K it is seen by the dog which starts to head towards it along the first line. When the dog reaches point E on the first line it notices that the rabbit has continued to run and is then at point L. The dog changes course and heads towards L along line 2. The rabbit continues, the dog changes course and the whole process is repeated over and over again. Make all the lengths RK, KL, DE, EF etc equal.

Here is another possibility:



In this case the dog has started at the centre of a circle and the rabbit is running around the circumference.

In the first case as the distances RK, KL, DE etc were all taken as equal, the dog and the rabbit were each covering equal distances in the same time.

In the second case the dog has been made to run at only half the speed of the rabbit.

Does the dog ever catch the rabbit?

Try drawing the first example on a large sheet of paper so that you can make the hedge as long as you like.

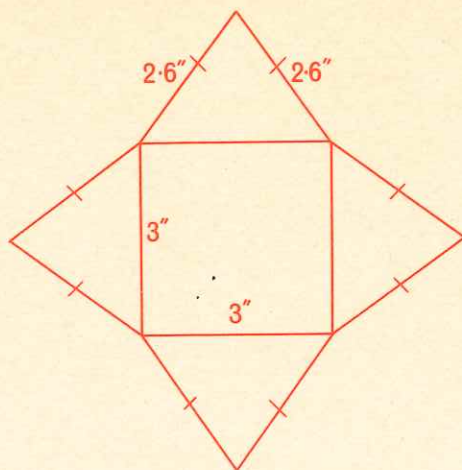
See what other patterns you can produce by this method. For example try having the rabbit run around the perimeter of a square field and start the dog either at the centre or at a corner opposite to the rabbit. Vary the speed so that sometimes the rabbit runs faster and sometimes slower than the dog.

A pair of dividers is useful for this work.

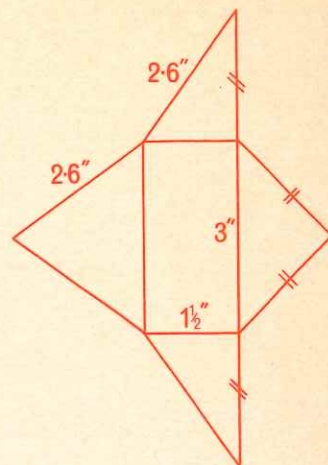
The Volume of a Pyramid

Part 1

Use manilla paper to make:

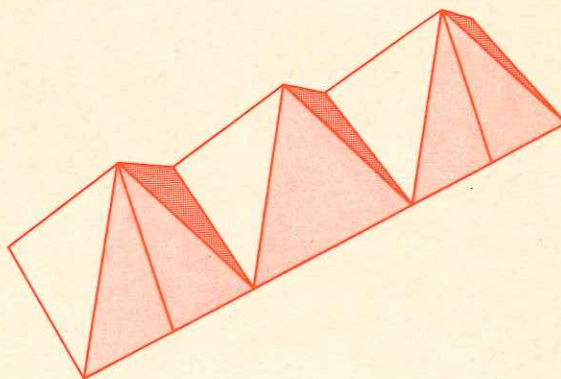


1 of these needed



4 of these needed

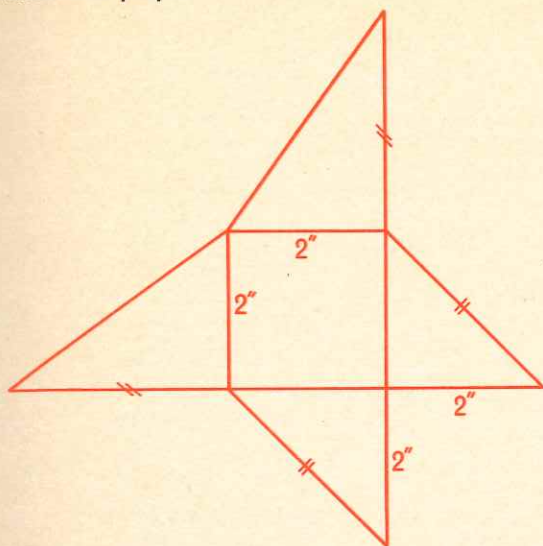
Cut out, fold and stick together to make five pyramids. These five pyramids can be arranged so that three equal, identical pyramids are produced and they can also all be put together to make a cuboid measuring $3 \times 3 \times 1\frac{1}{2}$ inches. Suitably hinged with sellotape the model will fold from three pyramids into a cuboid and vice-versa.



Since the three pyramids make a cuboid, you should be able to deduce the volume of one of the pyramids.

Part 2

To make this model cut *three* of the following net from manilla paper:



Each of these will assemble to make a pyramid.

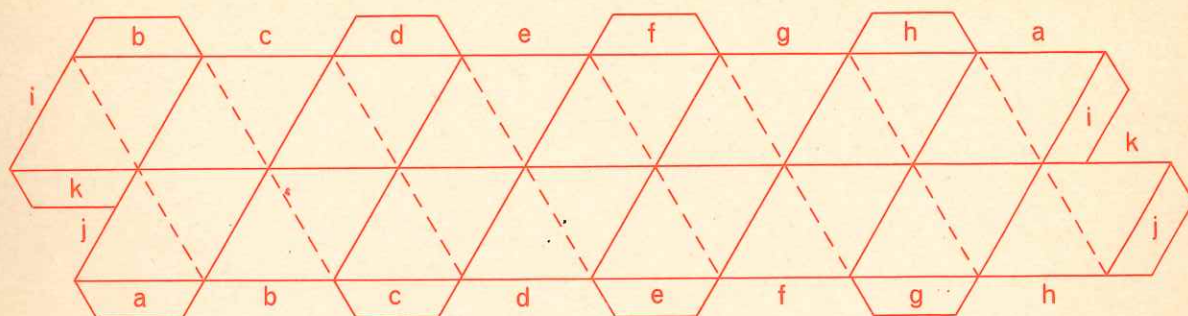
The three pyramids will go together to make a cube of side 2 inches.

As before, three pyramids make a cube and we can therefore calculate the volume of the pyramid.

These three pyramids can also be hinged so that they can be folded backwards and forwards.

A Ring of Pyramids

Draw this net of equilateral triangles with tabs on thin card or manilla paper. A side of $1\frac{1}{2}$ or 2 inches would be suitable.



Cut out and lightly score all lines, the solid ones on the front and the broken lines on the back.

Fold and crease and join tabs to edges with the same letter.

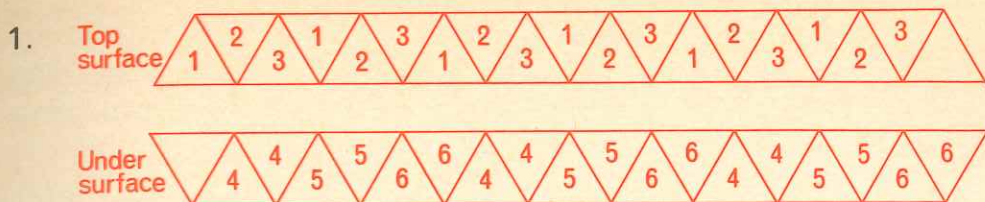
The finished model can be regarded as eight triangular based pyramids (regular tetrahedra) which are joined along some edges.

This model will turn round and round for ever and can be said to give the appearance of a smoke ring rising in the air.

Faces can be painted different colours to create attractive patterns and interesting effects can also be achieved by compressing the ring so that faces come together.

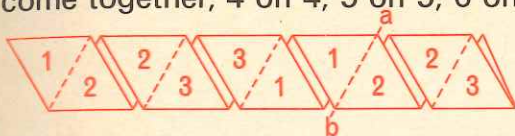
Longer strings of pyramids can be made simply by making the net longer, the figure with ten pyramids is particularly attractive and if twenty-two or more pyramids are made the ring can have a knot in it.

Hexahexaflexagon

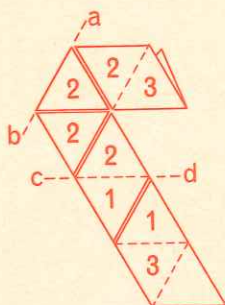


Use cartridge paper and cut out a strip of 19 equilateral triangles, number its surfaces as shown above.

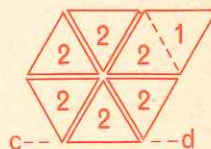
2. Fold the strip so that the same under surface numbers come together, 4 on 4, 5 on 5, 6 on 6 and so on.



3. Fold back along the line ab.



4. Fold back on the line cd. Turn the blank triangle under and paste it to the corresponding blank triangle on the other side of the strip.



If you have done the job properly the triangles on one face of the hexagon will all be numbered 1 and on the other face all numbered 2. If you pinch one corner while pushing the opposite corner it can be made to flex and change face.

The model can be made to look more attractive if the faces are coloured, all with the same number having the same colour.

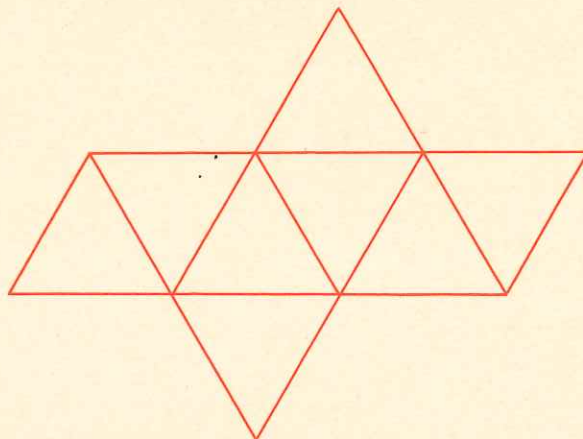
Having flexed from one corner do not move on directly to try another corner, sometimes the same corner will flex more than once.

Keep flexing until all faces have been made to appear, some are harder to obtain than others.

Polyhedra Nets

The five regular solids have been mentioned in other parts of these booklets and the nets of the tetrahedron and cube have been dealt with. Here are the nets of the three remaining solids.

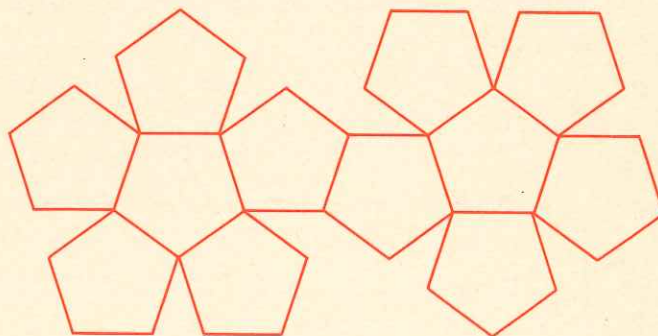
1. The Octahedron



Can you think of any other nets for this model?

2. The Dodecahedron

There is a way of constructing a pentagon with ruler and compasses only but it is involved and the sensible way to make this net is to use a protractor and measure the angles which are 108 degrees.



There are other possible nets for this as well.

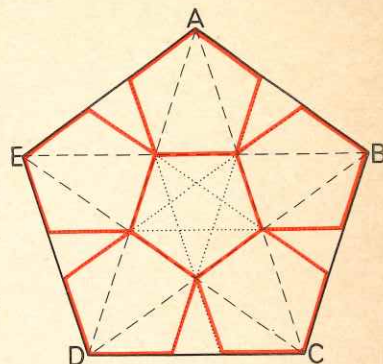
Here is a quick, accurate and interesting way of producing half the net of a dodecahedron.

Draw the pentagon ABCDE and make a five-pointed star (pentagram) inside it by joining A to C, C to E, E to B, B to D and D to A.

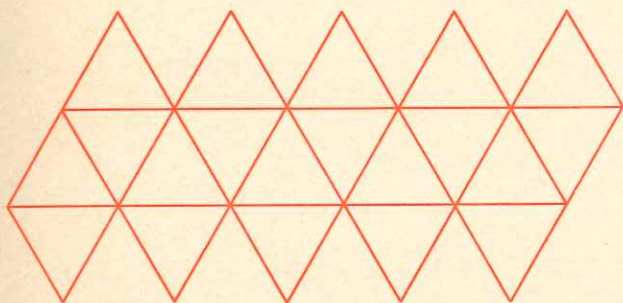
The centre of this star will now be another pentagon but upside down to the original one.

Draw another pentagram inside the small pentagon and produce its sides so that they meet the original figure.

You should now be able to pick out the half-net as shown in red above.



3. The Icosahedron



Isometric graph paper is very useful for this sort of net.

There are other possibilities.

The material to use depends on the size of model required, for most purposes good quality cartridge paper does quite well.

If tabs are required then it is left for you to decide where to put them.

Faces and Edges

Equipment: 7 strips of paper about 8 inches long and 1 inch wide. Scissors, sellotape.

Part 1

1. Take one strip and check on the number of faces and edges. If you put a finger on the edge you will notice that you can go right round and back to where you started without crossing a face. This shows that there is only one edge.

To get from one face to another you have to cross an edge. This shows that there are two faces.

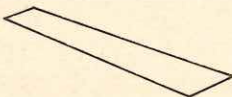
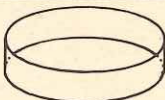
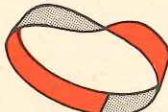
2. Take the next strip and join the ends together to form a loop. Check on the number of faces and edges now.

3. Hold the next piece by its ends, give one end a twist through two right angles and join the ends together. You will find it harder to check on faces and edges this time.

Check on edges with your finger as before but make a pencil mark to show where you start.

To check the faces it is advisable to shade a face with a pencil.

List your results in this table:

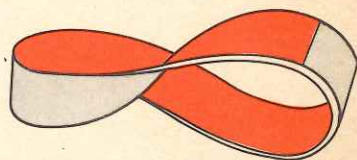
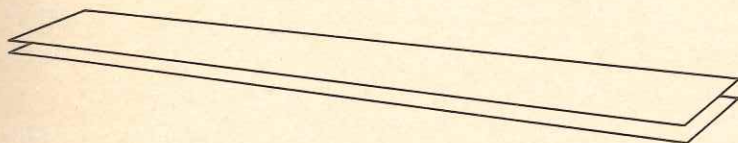
shape	name	faces	edges
	strip		
	band		
	band with half twist		

You will discover that the band with a half twist has unusual properties. It is called a Moebius band after a German mathematician of that name who was born in 1790.

Make another Moebius band and cut all round the middle of the strip with a pair of scissors. Try and guess what is going to happen before you get to the end. Describe your result in terms of faces and edges.

Make yet one more Moebius band and this time cut all the way round about a third of the distance from one edge. You will have to go all the way round twice before you meet up with the place you started. As before try and work out what is going to happen, you will probably get a surprise this time. Describe your result.

Part 2

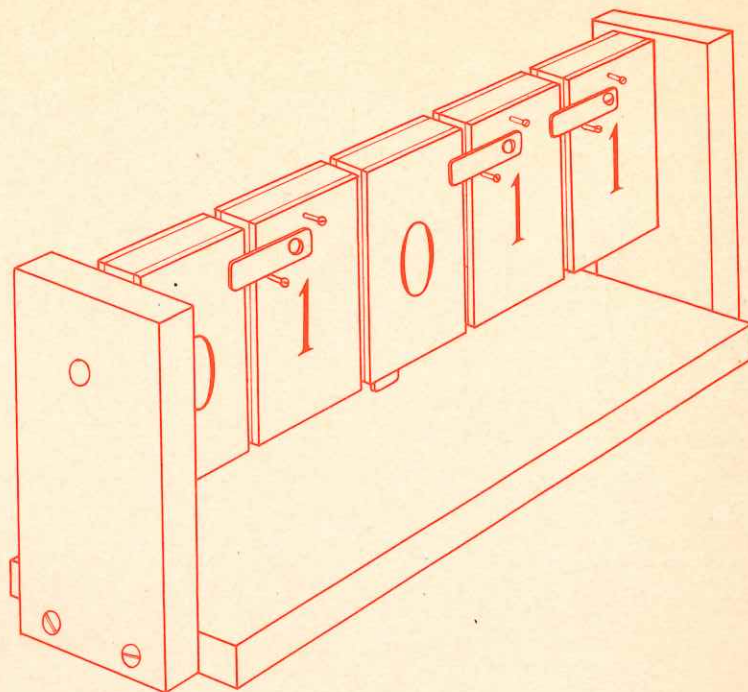


The double Moebius band is made by placing two strips together, give both a half twist and join the ends together.

This appears to be just two bands, one on top of the other, put your finger inside the bands and you can run it all the way round and return to place you started. An ant crawling between the bands could go round and round for ever as if it was walking between the surfaces of two separate bands where one was the ceiling and the other the floor. However, if the ant made a mark on the floor and circled until it reached the mark again it would be found on the ceiling and two complete circuits would be needed to find it on the floor again. It is not two bands at all and you can easily show this by holding one piece and shaking, and it becomes one large band. It is not so easy to put back again!

There are other interesting models to be made by twisting strips, but it soon becomes very involved.

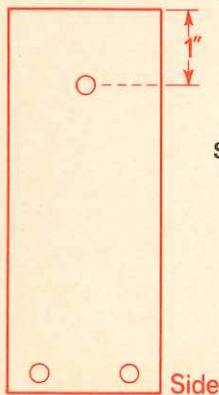
Plans for a Binary Adder



A 'Flip-flop' binary adding device which can be made with the simplest of tools. The five numbered blocks are slotted and rest on a piece of rod or dowelling. To operate you gently push the block at the top this causes it to rotate and then topple which makes sure that the carry arm is in position.

Instructions

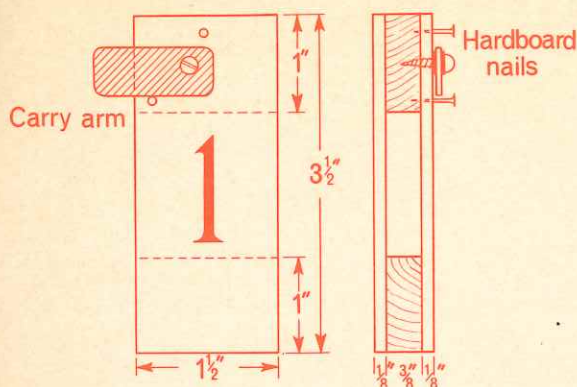
Base, rectangle $8 \times 3 \times \frac{1}{2}$ inch
sides, cut two, $5 \times 2 \times \frac{1}{2}$ inch



Drill $\frac{1}{4}$ inch diameter hole 1 inch from the top of both sides.

Drill 2 holes and countersink to take wood screws.
Screw and glue the sides to each end of the base.

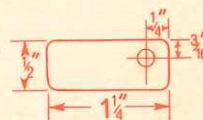
Detail of number block (half size).



For each block $\frac{1}{8}$ inch hardboard, two off, $3\frac{1}{2} \times 1\frac{1}{2}$ inches softwood inserts, two off, $1\frac{1}{2} \times 1 \times \frac{3}{8}$ inches.

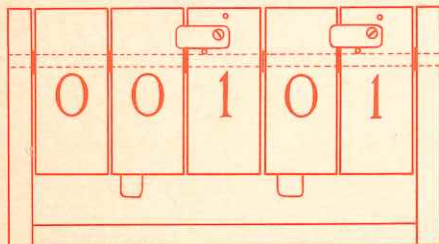
Make up with panel pins and avoid putting them too near to the edge in case of splitting.

Carry-arm. The positioning of this is most important. Make from a piece of $\frac{1}{2}$ inch wide thin metal strip, a piece of meccano would do if one side were filed away so that the screw hole is off-centre. Loosely screw to the block in the position shown above and put a small washer each side. Put in hardboard pins which act as stops. The left-hand number block does not need a carry-arm.



Assembly. Feed a 9 inch length of $\frac{1}{4}$ inch diameter dowelling through the hole in one of the sides, put on a large washer then a block followed by another washer then another block until all five blocks are on; finally one more washer and push the dowelling into the other end.

Use. Adding has been described above, to subtract you add the complement.



Number in the Window

Part 1

YES

1	3	cut out	
5	7		
9	11		
13	15		
cut out		17	19
		21	23
		25	27
		29	31

ON

①

YES

2	3																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																	
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YES

4	5
6	7
12	13
14	15

cut out

20	21
22	23
28	29
30	31

cut out

ON

YES

8	9
10	11
12	13
14	15

cut out	24	25
	26	27
	28	29
	30	31

ON

4

YES

NO

16 17
18 19
20 21
22 23

24 25
26 27
28 29
30 31

5

← front

back →

16	17	6	4	2	
18	19				
20	21	7	5	3	1
22	23				
14	12	10	8	24	25
				26	27
				28	29
				30	31

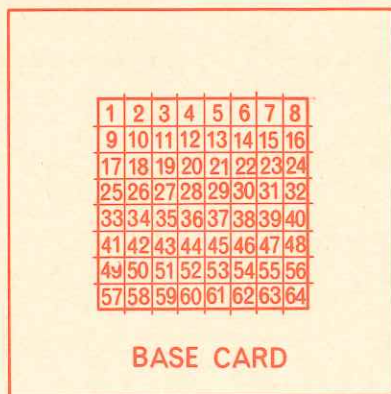
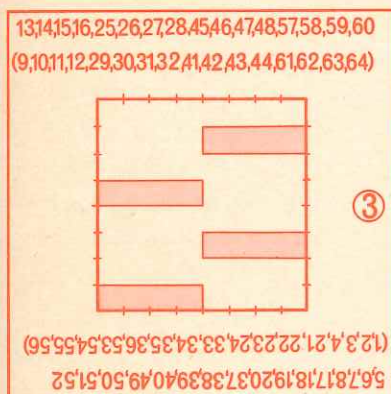
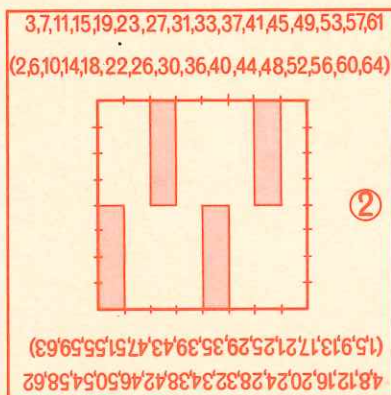
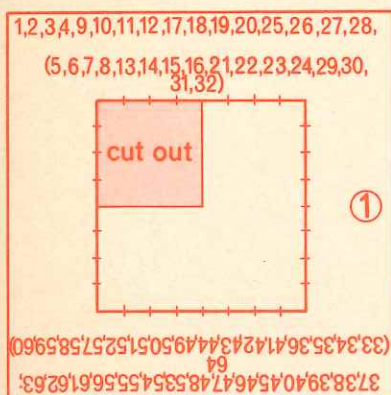
Use five squares of card each 5 inch by 5 inch, make the small boxes 1 inch long and $\frac{1}{2}$ inch wide and the margins $\frac{1}{2}$ inch wide.

Ask a person to pick a number from 1 to 31 but not to tell you. Show him card No. 1 and ask if his number is on the card, if the answer is 'Yes' place the card flat on a table with the word 'Yes' at the top, if the answer is 'No' place the card so that 'No' is at the top. Work through all five cards this way, stacking them on top of each other. Finally pick the whole pack up together and show your person the undersurface; the number he chose should be showing 'through the window'.

This trick depends upon the binary scale, to help discover how it works you should write the numbers found on each card in the scale of two keeping them underneath each other.

Part 2

Here is a similar trick which depends upon the scale of four.

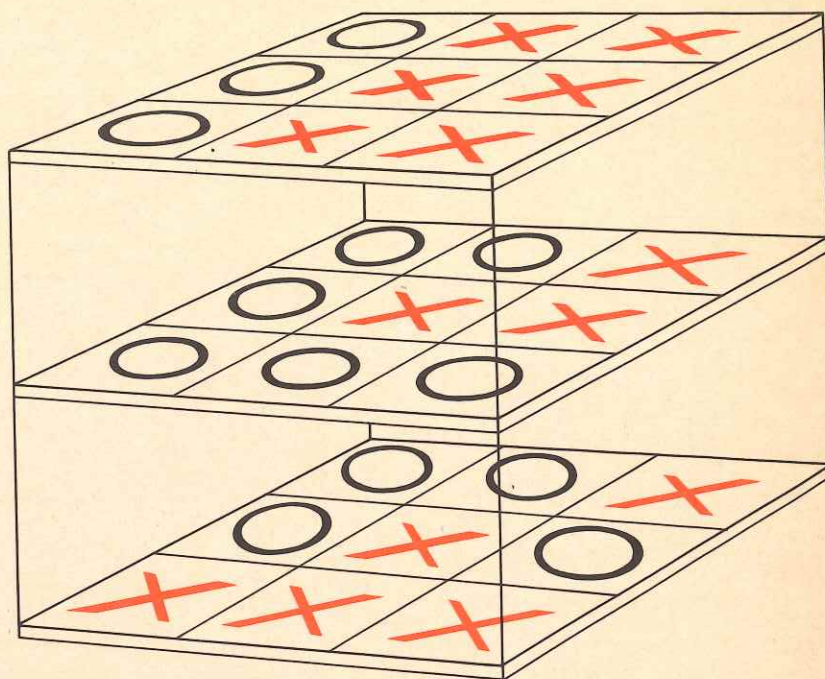


Cut out the shaded parts. All the numbers in brackets must be written on the reverse side of each card.

You can find out how to use this one yourself.

Three-dimensional Noughts and Crosses

This is played in just the same way as ordinary noughts and crosses except that you can play on any one of three levels and you carry on until all 27 spaces are filled.



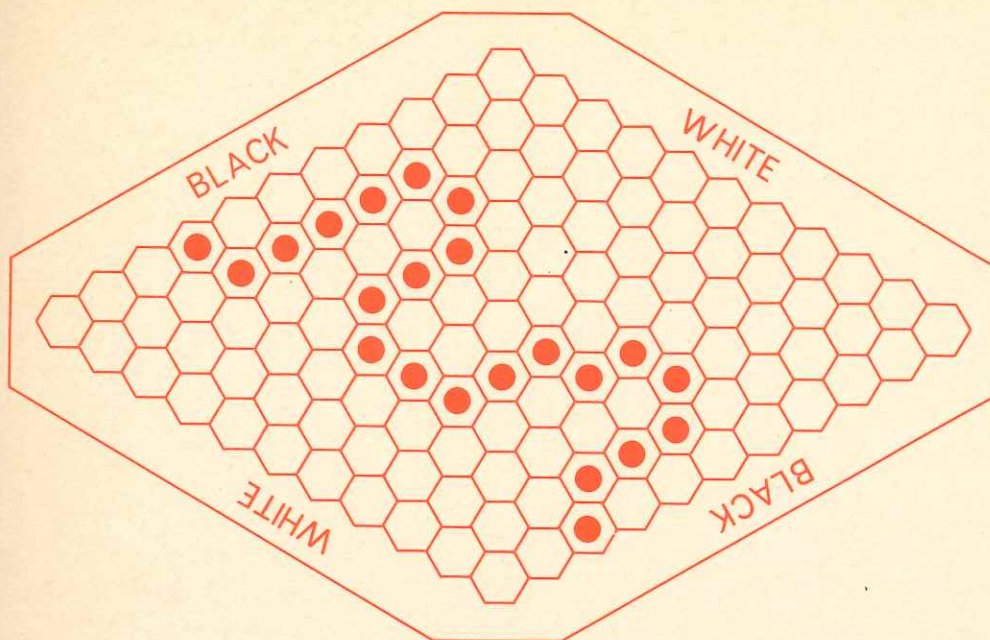
If it is your go first which is the best square to occupy?

Winning lines can be either in a horizontal or vertical plane or any three in a diagonal line. How many lines has each of the opponents in the above game?

The best material to use to make this game is perspex, but thinner acetate sheet will do. It can be made with wood having holes bored in it for coloured marbles to rest in, but it is not so easy to count up winning lines.

The Game of Hex

This is a new mathematical game played on a diamond shaped board covered in regular hexagons.



The number of hexagons may vary, but eleven along each edge is the usual number. One pair of opposite sides is called 'Black' and the other pair 'White', the hexagons at the four corners belong to either side. A supply of black pieces and white pieces is needed.

Players alternatively put a piece on any hexagon which is not already occupied. Each player tries to complete an unbroken chain of pieces of his own colour between his two sides. The chain need not be straight, an example of a winning black chain is shown in the diagram.

Can this game end in stalemate?

To make a Hex board isometric paper is very useful and it is possible to obtain paper with a hexagonal grid on it, however the easiest way to play is to cut a diamond section from a piece of half inch mesh chicken wire and lay it on a piece of paper, one player writing in noughts and the other crosses.

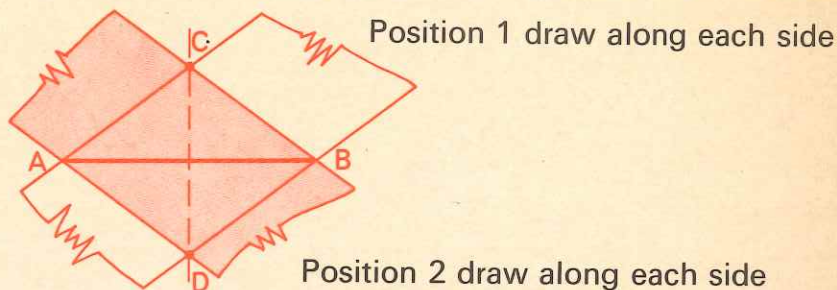
Geometry without Instruments

We have already seen how to make many geometrical figures without the usual instruments of compass and protractor, here is another technique.

Part 1 Parallel Strip Constructions

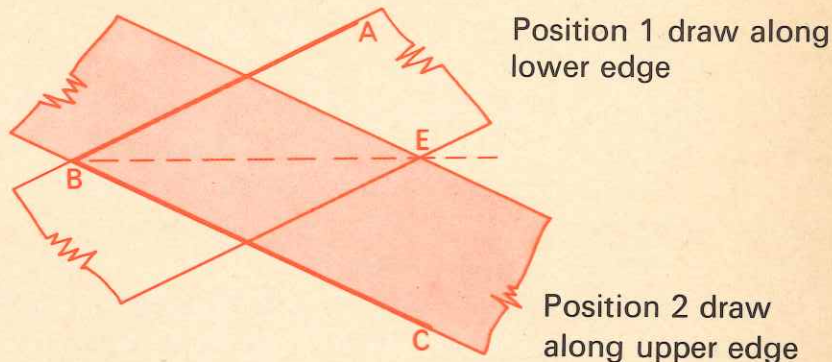
All that is needed for this work is a strip with parallel sides such as a thin lathe or a plain ruler.

(a) To construct the perpendicular bisector of a line AB:



CD is the required line

(b) To bisect the angle ABC:



BE is the required bisector

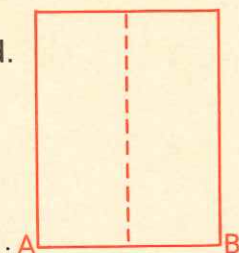
(c) You should now try to discover how to do other constructions yourself, for example, drop a perpendicular from a point to a line; erect a perpendicular at a point on a line.

More Paper Folding

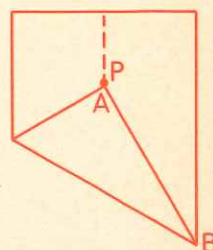
Part 1 To make an equilateral triangle

Start with a rectangular sheet of paper.

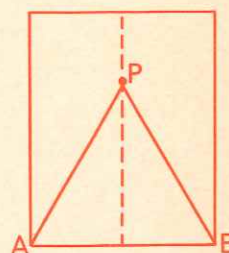
Crease down the middle and unfold.



Lift corner A over until it reaches the crease, mark point P where it comes.



Flatten out again. Then triangle APB is equilateral.

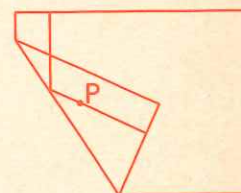
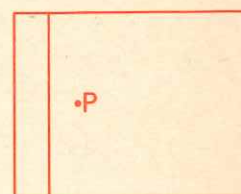


Part 2 To fold the envelope of a parabola

The only really practical way of doing this is to use tracing paper, not only does it make it possible to see what is happening, but the process of creasing breaks the surface and leaves a fine white visible line which does not need pencilling over.

Mark a line near one end of the tracing paper and put a point P about $\frac{1}{2}$ inch away from it.

Fold the paper so that the line covers the point P, crease, unfold and repeat for many positions until the curve is apparent.





Although π is easily defined as the ratio of the circumference of a circle to its diameter it is difficult to find its value, in fact it is impossible to find its exact value in the ordinary number system.

π is calculated as a decimal from various series but the decimal does not end or recur, different orders of numbers keep arising and as you are working out the answer you can never say what number is coming next.

Modern computers can calculate π to thousands of places of decimals.

Here is the value of π correct to 22 decimal places:

3.1415926535897932384626

but a value of 3.14 is close enough for most purposes.

An Experimental Approach to π

Take a large sheet of drawing paper and rule a series of parallel lines 2 inches apart right across it.

Cut ten used matches so that they are each one inch long, hold them about a foot above the middle of the ruled paper and lightly toss them up so that they fall and scatter over the lines.

Count the number of matches that have either crossed or touched a line and record your results.

Drop number	1	2	3	4	5	6	7	8
No. crossing or touching								

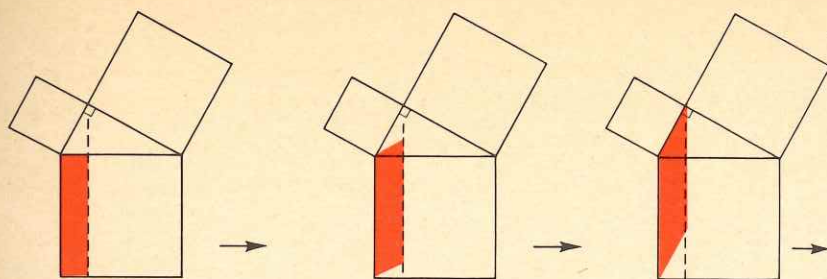
To get a worthwhile result you should do at least 100 drops, so here is a case where many groups can do the same experiment and pool results.

Finally add up the total number of sticks dropped and the total number which either crossed or touched a line then the division sum:

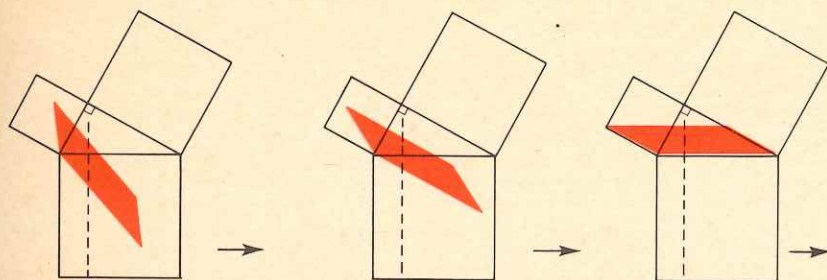
$$\frac{\text{Total dropped}}{\text{Total crossing or touching}}$$
 is approximately equal to π

See how close you can get to 3.14, the more you do the more accurate your answer should be.

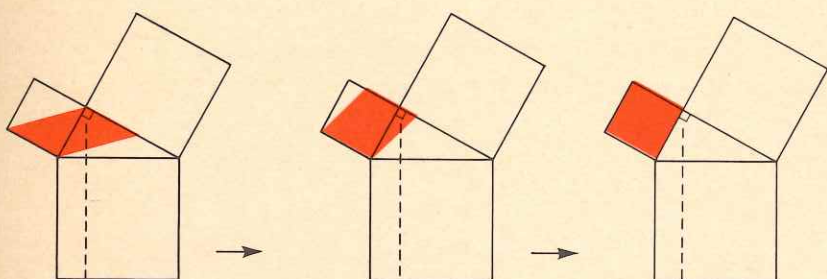
A 'Riffle' Book



shear



rotate



shear

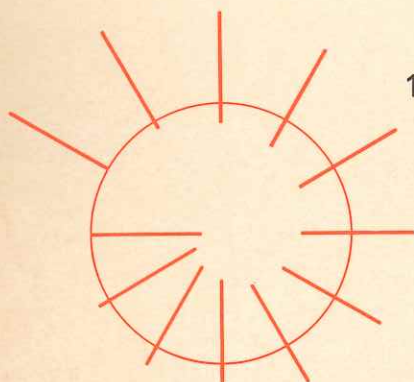
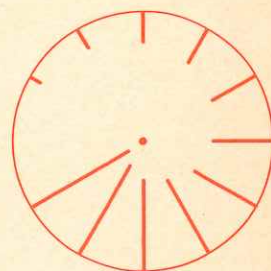
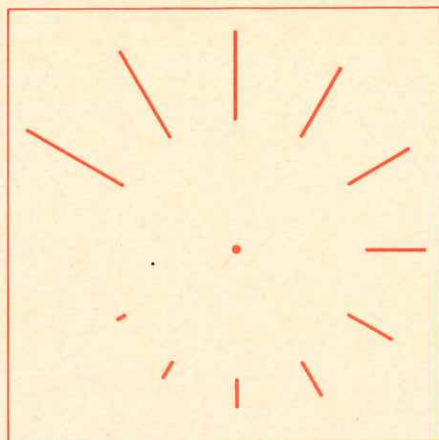
The left-hand rectangle of the bottom square is transformed into the small square. Exactly the same procedure will transform the right-hand rectangle into the top right square. This shows that the largest square is equal to the sum of the other two squares and is Pythagoras' Theorem once again.

Duplicate a large number of outlines and put in some more intermediate positions, when the pages are fixed together and 'riffled' with the thumb the appearance of motion is obtained.

This could be treated as a class activity. Make a template for the rotating part.

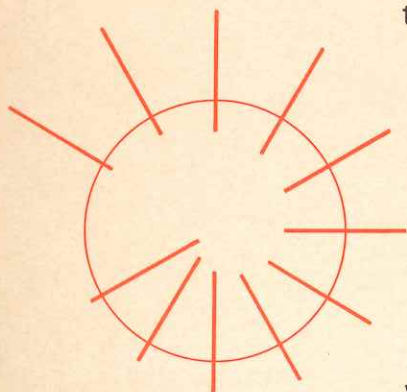
Illusions

Here is an accurate diagram for you to trace on card. Put a paper clip through the centre so that the disc revolves.



12 lines

turn the centre anti-clockwise and now only 11 lines.

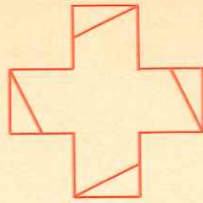


Where has the other line gone?

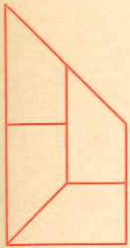
Answers

Page 6. The Greek Cross

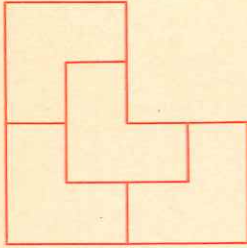
Part 1. The Hindu Problem.



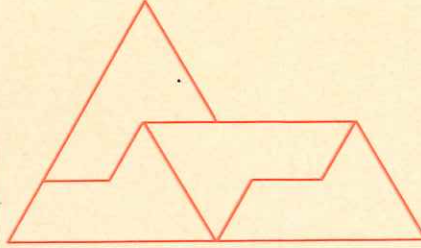
Page 8. 'Reptiles'



Part 1



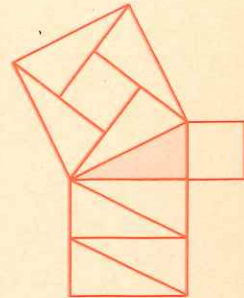
Part 2



Part 3

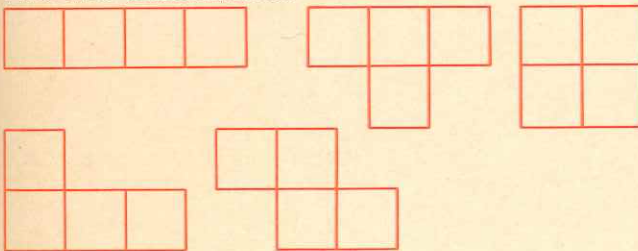
Page 12. Pythagoras' Theorem

The shapes on the palace floor were right-angled isosceles triangles, the angles were 90° , 45° , 45° .

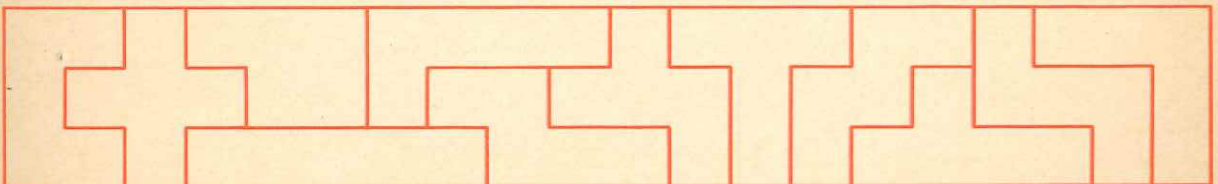


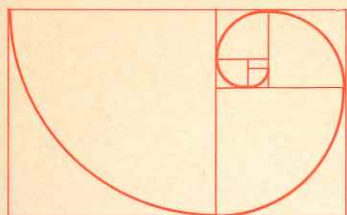
Page 14. Polyominoes

The five tetrominoes



Here is one of the 3×20 patterns



Page 20. Rectangles

Part 1. If diagonals are equal it is a rectangle.

Part 2. Here is a hidden spiral, others can be found.

Part 3. The last two columns should be the same in each case $L^2 + B^2 = D^2$

This is an example of Pythagoras' Theorem.

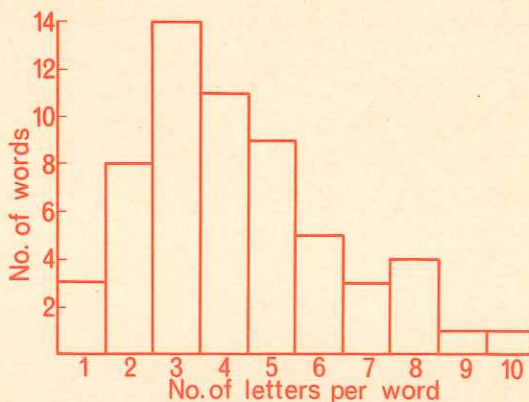
Page 22. Codes

Part 3. Here is the complete count:

A	20	J	0	S	17
B	2	K	4	T	29
C	5	L	7	U	11
D	13	M	4	V	2
E	31	N	19	W	7
F	3	O	16	X	0
G	5	P	2	Y	3
H	20	Q	1	Z	0
I	18	R	10		

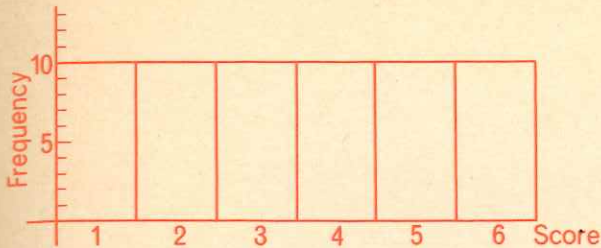
Part 3. The top six:

E	31
T	29
A	20
H	
N	19
I	18

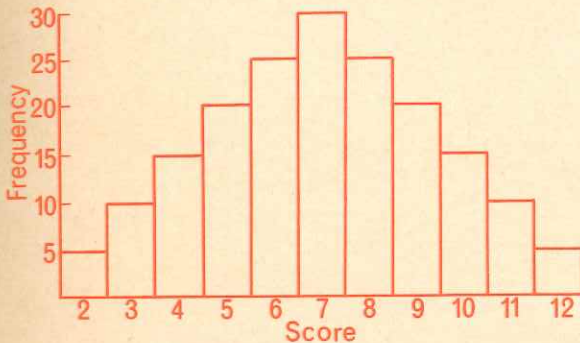


Page 24. Dice

Part 1. A 'perfect' set of results would give 10 of each score.



Part 2. A 'perfect' set of results would give a graph going up and down in steps of 5.

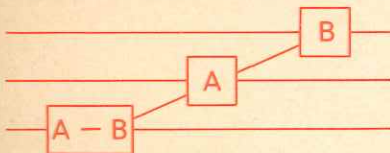


Part 3. The 'perfect' or theoretical results for three dice:

Score	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
No. of times out of 216	1	3	6	10	15	21	25	27	27	25	21	15	10	6	3	1

Page 28. Devices for Addition.

Part 2. A flow-chart for subtraction.



Part 3. A base 4 nomogram for addition:



Page 30. Napier's Bones.

Part 1. (a) 68, (b) 532, (c) 225, (d) 1456, (e) 264,
(f) 1368, (g) 3082.

Part 2. The set of base 8 bones.

1	0 1 2 3 4 5 6 7
2	0 2 4 6 1 0 2 4 1 6
3	0 3 6 1 1 4 1 7 2 2 5
4	0 4 1 0 1 4 2 0 2 4 3 0 3 4
5	0 5 1 2 1 7 2 4 3 1 3 6 4 3
6	0 6 1 4 2 2 3 0 3 6 4 4 5 2
7	0 7 1 6 2 5 3 4 4 3 5 2 6 1

1

0 1

This is all there would be in a set of base 2 bones!

Page 40. Volume of a Pyramid.

Part 1. Volume of cuboid $13\frac{1}{2}$ cubic inches

Volume of pyramid $4\frac{1}{2}$ cubic inches

Part 2. Volume of cube 8 cubic inches

Volume of pyramid $2\frac{2}{3}$ cubic inches

Page 46. Faces and Edges.

Name	F	V
Strip	2	1
Band	2	2
Half twist	1	1

Page 52. 3-D Noughts and Crosses.

Cross has 9 winning lines.

Nought has 6 winning lines.

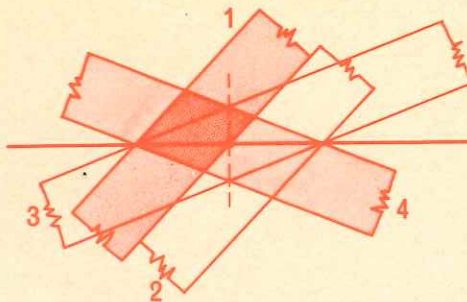
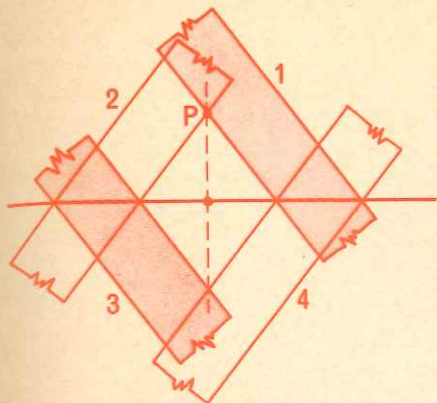
Page 53. The Game of Hex.

The game can never end in stalemate, someone has to win.

Page 54. Geometry without Instruments.

To drop a perpendicular from a point to a line.

To erect a perpendicular at a point on a line.





HOUGHTON MIFFLIN

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